Countercyclical CoCos

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Abstract

We present a new variant of the contingent convertible capital instrument (CoCos), countercyclical CoCos (CC-CoCos), to enhance financial stability and resilience. Following the countercyclical capital buffer framework of Basel III, we show that banks' capital can be increased by converting CC-CoCos into common equity and writing down their principal during periods of credit expansion. The potential transfer of risk to taxpayers and government bailouts is therefore reduced. Using credit-to-GDP ratio, the primary indicator suggested by the Basel Committee for Banking Supervision, for triggering the conversion of CC-CoCos, it avoids the death spiral risk found in conventional CoCos. In addition, it mitigates the problems of opacity, manipulation, and multiple pricing related to accounting or market value-based triggers. Finally, the value of conversion terms and write-down of CC-CoCos provide a clear cost to investors and therefore an understanding of the risk and return tradeoff.

Keywords: Basel III; Contingent convertible capital; Countercyclical capital buffers; Credit-to-GDP ratio.

JEL Classification: E44, G21, and G28.

1. Introduction

In response to the financial crisis of 2007–2008, the Dodd-Frank Wall Street Reform and Consumer Protection Act (Dodd-Frank Act) was legislated in 2010 to overhaul the US financial regulatory system. Among the regulatory reforms in the Dodd-Frank Act, Section 115(c) requires the Financial Stability Oversight Council (FSOC) to evaluate the use of contingent convertible capital to enhance the safety and soundness of issuing banks, promote financial stability in the sector, and thereby, reduce risks to government and taxpayers. Contingent convertible capital, commonly known as CoCos, is a hybrid capital instrument used to absorb losses when the issuer's bank capital falls below a threshold. It can also be used to satisfy regulatory capital requirements by converting them into common equity or writing down the principal. Flannery (2005) suggests that banks issue CoCos to impose market discipline on banks and reduce a bank's expected cost of bankruptcy or potential bailouts. Similar proposals by the Basel Committee on Banking Supervision (BCBS (2010a)) and subsequent approvals by European regulators (e.g., European Banking Authority (2011), European Commission (2011)) have led to a large increase in CoCos issued by European banks.[1,2]

In this study, we develop a new variant of CoCos, countercyclical CoCos (CC-CoCos), according to the countercyclical capital buffer (CCB) framework of Basel III, which require banks to build up capital buffers during periods of credit expansion. BCBS suggests that the credit-to-GDP ratio (credit/GDP) can be as a primary indicator to signal the build-up of capital buffers when

[1] Global systemically important banks are required to meet their additional loss absorbency requirements using common equity tier-1 only. However, CoCos can be used to meet higher national capital requirements rather than the Basel capital requirements.

[2] According to CreditSights, European banks have issued CoCos worth more than €100 billion since 2012.

it exceeds a chosen threshold above its long-term trend. Drehmann and Juselius (2014) report that credit/GDP is the single best macroeconomic indicator that provides the predictability and stability for the implementation of the macro-prudential policy. For the application of our new security design, CC-CoCos are converted into common equity during expansion periods to increase regulatory capital when credit/GDP exceeds the threshold of its long-term trend. The timing of the CC-CoCos conversion therefore contrasts to that of the conventional CoCos which takes place during economic contraction periods.

The unique feature of converting CC-CoCos during the expansion phase of the credit cycles has certain advantages. First, consistent with the spirit of CCB schemes and macro-prudential regulations, banks can accumulate their capital buffers to mitigate the build-up of systemic risk in the financial sector during periods of excessive credit, as Schularick and Taylor (2012) argue that it is a precursor of a financial crisis. The accumulation of capital buffers also has the benefit of mitigating the procyclical characteristics of the current Basel risk-based capital requirements (Lee et al. (2012), Repullo and Suarez (2013), Behn et al. (2016)). By converting CC-CoCos into core equity during the expansion phase, banks can strengthen their regulatory capital without the need to seek external equity. Guidara et al. (2013) report that Canadian banks with capital buffers larger than those of their foreign counterparts weathered the recent financial crisis relatively well. It follows that the design and implementation of CC-CoCos should enhance bank stability with additional capital buffer to withstand a severe credit downturn.

Second, this inherent feature of CC-CoCos is likely to avoid the death spiral effect, an unintended consequence, associated with CoCos. Hillion and Vermaelen (2004) argue that as CoCos are converted into shares when banks fail to satisfy their capital standards during bad times, investors could hedge against the equity exposure due to share dilution by taking a short position on the banks' stocks. This exacerbates the downward pressure on falling stock prices. Although safety features such as multiple triggers can be added to standard CoCos to alleviate the death spiral risk (Spiegeleer and Schoutens (2013)), the negative signal of conversion or panic runs may continue to push stock prices down. Hoshi and Kashyap (2010) report that during the 1990s, Japanese banks were reluctant to seek government funds for fear of admitting larger future losses or the inability of raising external funds. The conversion of CC-CoCos during periods of high credit growth tend to mitigate the impacts of negative signaling and share dilution. Investors are also less likely to take a short position when banks are performing well.

The choice of credit/GDP, a macro-based trigger, in the CCB schemes to convert CC-CoCos into equity further overcomes some problems related to micro-based triggers. For example, CoCos with accounting-value triggers include those based on book value of equity (Glasserman and Nouri (2012), Wilkens and Bethke (2014)), cash flows (Koziol and Lawrenz (2012)), and book value of assets (Berg and Kaserer (2015)). These triggers may not be reliable since they are susceptible to accounting rules that could deviate belatedly from market value. Banks can also underestimate risk-weighted assets based on Basel's the internal-ratings-based model to lower capital requirements (Mariathasan and Merrouche (2014), Plosser and Santos (2015). Duffie (2009) reports that Citigroup, which was bailed out by the government, had a Tier 1 accounting capital ratio of at least 7% throughout the recent financial crisis. Similarly, De Groen (2011) finds that Dexia Group had a Tier 1 ratio of 10% in 2010, the same year in which it was rescued by the government. Therefore, the lack of reliability and consistency of accounting-value triggers casts doubt on the timely conversion of CoCos. In contrast, conversions based on credit/GDP avoid these accounting problems.

CoCos designed with market value triggers face different challenges. Instead of banks distorting accounting-based signals, speculators can manipulate market-based signals by shorting banks' stocks to force a conversion to further lower stock prices (Hillion and Vermaelen (2004)). In a severe scenario, it leads to the death spiral effect, as discussed earlier. The opacity in bank assets and complexity in bank risks also make it difficult to value bank equity. Furthermore, there is an on-going issue regarding the market pricing of CoCos. Sundaresan and Wang (2015) highlight that CoCos with market value triggers could end up with either multiple or no pricing. They show that conversion terms that are advantageous to CoCos holders lead to multiple equilibria in stock prices; however, those advantageous to existing shareholders lack stock price equilibrium. Nevertheless, Glasserman and Nouri (2016) prove there is a unique equilibrium stock price when conversion terms are advantageous to CoCos holders. Pennacchi and Tchistyi (2016) further show that CoCos with perpetuities under normal conditions can have a unique equilibrium stock price when conversion terms or write-downs favor existing shareholders.

To address aforementioned concerns related to market value triggers, additional features have been proposed on CoCos. McDonald (2013) suggests CoCos with a dual price trigger, one that is based on the bank's stock price and the other on the value of a financial institution index. Calomiris and Herring (2013) argue that conversions should be based on a moving average of the ratio of quasi market to equity value. Pennacchi et al. (2014) introduce a call option enhanced reverse convertible (COERC) that can be triggered by the market value of total capital but provides existing shareholders an option to buy back shares from COERC holders at the par value to reduce share dilution. Bulow and Klemperer (2015) propose equity recourse notes that convert interest or principal payment into equity when the stock price is below the threshold on the payment due date. CC-CoCos fundamentally differ from these improved CoCos with the use of a system-wide trigger that should be free from market speculation, death spiral, and potential multiple pricing effects. It therefore gets away with the complexities of additional features associated with market triggers. It is also interesting to note that using credit/GDP for conversions is consistent with Section 115(c) of the Dodd-Frank Act, which suggests that macroeconomic based triggers should be considered for CoCos conversion.

Another problem with CoCos is that the terms of a write-down (WD) and conversion condition differ among issuers. It can therefore be difficult to value them and poses a hidden risk to investors. For example, CoCos may include a WD clause to reduce the principal when regulatory capital is below a given threshold (Attaoui and Poncet (2015)). Banks may therefore choose this clause rather than convert CoCos to common equity to prevent share dilution. Contrary to the risk and return tradeoff, Avdjiev et al. (2013) find that CoCos with partial WD at a lower threshold of regulatory capital exhibit a higher yield to maturity than those with a higher threshold. They suggest that this pricing anomaly can be confounded by regulators' discretionary decisions or the point of non-viability clause commonly found in these CoCos.

In addition, most CoCos are perpetual bonds with coupon payments and are expected to be called back in the future. Similar to dividends, banks may choose to forego coupon payments, especially when bank capital is low. The risks to investors at the issuer's discretion on a call back or dividend payout are therefore difficult to evaluate. Regulators may also restrict banks' conversion of CoCos if the costs of new issues are higher. In sum, these unknown uncertainties may create a risk-return relationship that even sophisticated investors may not understand. To partly address such concerns, the Financial Conduct Authority in UK restricts banks' distribution of CoCos to retail investors.

Our design of CC-CoCos does not suffer from these hidden complexities since WD terms are well specified and, more importantly, conversion triggers are directly linked to the thresholds outlined in the CCB framework. For instance, as credit/GDP exceeds its long-term trend above the lower threshold (e.g., two percent), the conversion into common equity is triggered. As credit/GDP continues to increase above its long-term trend, more CC-CoCos are converted into common equity on the basis of a pre-determined conversion rate until the higher threshold (e.g., ten percent), where a full conversion is complete. With the specifications of the WD and conversion terms, we can value CC-CoCos by first, estimating the distribution of the credit/GDP relative to its long-term trend to identify the probability of the WD and conversion trigger, and extent of conversion rate. Next, we follow Glasserman and Nouri’s (2012) valuation approach and determine the CC-CoCos spread, the difference between the coupon rates of a CC-CoCo and non-convertible bond to capture the risks associated with the WD and conversion probability, and conversion terms. This approach of pricing CC-CoCos provides investors with a clear understanding on their risk-return tradeoff.

We show that the term structure of the CC-CoCos spread can be positive or negative and hump shaped or monotonically increasing depending on the conversion probability, conversion price, and write-down terms. In particular, factors that increase the conversion probability are likely to lead to a higher spread in the shorter-term maturity and therefore, a hump shape over the term structure. Those lowering the probability tend to result in a normal yield curve, which is similar to risk-free bonds. A conversion price that is greater than the stock price or a higher write-down ratio also produces a hump shape over the term structure. In contrast, a conversion price that is less than the stock price is associated with an inverted hump shape or negative spread.

The remainder of this paper is organized as follows. Section 2 briefly discusses the CCB framework. Section 3 derives closed-form solutions for CC-CoCos and determines the CC-CoCos spread, which is the difference in the yields between CC-CoCos and non-convertible bonds. Section 4 presents certain numerical examples and model parameter analyses. Section 5 concludes the paper. Appendix 1 provides the derivations and proofs for the closed-form solutions for CC-CoCos' pricing.

2. Countercyclical Capital Buffers

BCBS (2010a) points out that the financial crisis of 2007–2008 taught regulators a profound lesson on extreme losses incurred in the financial sector subsequent to excessive credit growth. It highlights the importance of accumulating capital buffers to mitigate the systemic risk built up in the expansion phase of financial cycles. Accordingly, BCBS proposes the CCB framework that requires banks to increase capital buffers during expansion periods and release them during contraction periods. In particular, an indicator can be used to signal the build-up of capital buffers when it exceeds a chosen threshold of its long-term trend. Drehmann et al. (2010) and BCBS ((2010b), (2010c)) find that, among the seven indicators examined from 36 countries, credit-to-GDP ratio and deviation from its long-term trend (credit/GDP gap) should be used as a key indicator by regulatory authorities to determine the level of CCB across banks. The credit/GDP gap at time t can be expressed as

$$credit/GDP\ gap_t = (credit_t / GDP_t - trend_t), \quad (1)$$

where $credit_t$ denote credit of households and the private, non-financial corporate sector, including non-banks and lending from abroad; GDP_t is gross domestic product; and $trend_t$ is the long-term trend based on a one-sided Hodrick–Prescott (HP) filter.[3]

More specifically, BCBS (2010a) suggests that banks should start to accumulate capital buffers when the credit/GDP gap exceeds the lower threshold (L) of 2%, that is, at least two to three years prior to a financial crisis. Capital buffers should increase linearly with the credit/GDP gap until 2.5% of risk-weighted assets, when it reaches the higher threshold (H) of 10%. Figure 1 shows the relationship between the level of CCB and the credit/GDP gap, which be classified into the following three cases:

Case 1: If credit/GDP gap ≦ L => CCB is 0.

Case 2: If credit/GDP gap ≧ H => $\max CCB$.

Case 3: If L < credit/GDP < H => $\max CCB \times \frac{(\text{credit/GDP gap} - L)}{(H - L)}$.

[Insert Figure 1]

3. Pricing of CC-CoCos

To qualify as a CoCo, BCBS specifies that it must have the ability to absorb losses prior to accepting government assistance or if regulators consider it necessary to avoid bankruptcy. Loss

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[3] An HP filter can be used to separate trends from cycles in the credit-to-GDP data. Using a smoothing parameter (lambda(λ)) of 400,000 recommended by BCBS, the following trend can be obtained:

$$\min_{\{Trend\}_{t=1}^{T}} \sum_{t=1}^{T} (credit/GDP_t - Trend_t)^2 + \lambda \sum_{t=1}^{T} [(Trend_{t+1} - Trend_t) - (Trend_t - Trend_{t-1})]^2 .$$

absorption mechanisms should either be writing off CoCos or converting into common equity when bank capital falls below a threshold. Figure 2 depicts these two important features incorporated into CC-CoCos: a countercyclical trigger event and loss absorption requirements.

[Insert Figure 2]

3.1 Countercyclical Conversion Trigger

As a macro-based trigger in the CCB framework, the credit/GDP gap issues earning warning signals for banks to raise capital buffers two to five years before a crisis. We incorporate this key feature into CC-CoCos by first establishing the positive relationship between CC-CoCos' conversion rate ($C_{\{trigger\}}$) and the credit/GDP gap. The conversion rate is the proportion of the face value of CC-CoCos that is converted into common equity after the credit/GDP gap exceeds the lower threshold (L). Let N be the principal of CC-CoCos, t_n the issued date, and T the maturity date. Since banks may need to increase their capital buffers to the maximum level prior to the maturity date, we use the maximum credit/GDP gap, $M_{t_n}^{T}$, over the lifetime of CC-CoCos to determine the conversion rate. If $M_{t_n}^{T}$ exceeds the lower threshold (L), the face value of CC-CoCos will begin to proportionally convert into common equity. If $M_{t_n}^{T}$ reaches the higher threshold (H), the face value will fully convert into common equity. The conversion rate can be summarized as follows:

$$C_{\{trigger\}} = \begin{cases} 0 & \text{if } M_{t_n}^{T} \leq \mathrm{L} \\ \dfrac{M_{t_n}^{T} - L}{(H - L)} & \text{if } \mathrm{L} < M_{t_n}^{T} < H \\ 1 & \text{if } M_{t_n}^{T} \geq \mathrm{H}. \end{cases} \qquad (2)$$

In Eq. (2), the proportion of the face value of CC-CoCos converted to common equity, CFV, at maturity *T* is given by

$$CFV = NI_{\{trigger\}} = N \frac{[M_{t_n}^{T} - L]^{+} - [M_{t_n}^{T} - H]^{+}}{(H - L)}, \tag{3}$$

where $[M_{t_n}^{T} - L]^{+} = Max(M_{t_n}^{T} - L, 0)$ and $[M_{t_n}^{T} - H]^{+} = Max(M_{t_n}^{T} - H, 0)$

3.2 Loss absorption Mechanism

For the second feature of CC-CoCos, we design a loss absorption mechanism to satisfy the principal write-down and common equity conversion. Harris and Raviv (1985) define the conversion ratio as the number of converted shares of convertible bonds at maturity. If C_p is the conversion price and α is the write-down ratio, the CC-CoCos conversion ratio is

$$(1-\alpha)(\frac{NC_{\{trigger\}}}{C_p}) = (1-\alpha)(\frac{CFV}{C_p}). \tag{4}$$

Eq. (4) shows the number of shares that can be converted into common equity (CFV / C_p) at time *T* after the write-down ($1-\alpha$). If α = 75%, investors only receive 25% of the face value in common equity. A higher conversion price or write-down ratio would lead to a lower conversion ratio and reduce the dilution effect on pre-existing shareholders.

By multiplying Eq. (4) with the underlying stock price, S_T, at time *T*, we can obtain the loss absorption market value (LMV) available to cover future potential losses:

$$LMV = (1-\alpha)(\frac{S_T}{C_p})NC_{\{trigger\}} \tag{5}$$

If C_p is pre-specified as a percentage of S_T at the time of issuance, we can reduce the uncertainty in the amount of wealth transfer between CC-CoCos holders and pre-existing shareholders. Assuming $C_p = xS_T$ and substituting Eq. (3) into Eq. (5), LMV can be expressed as

$$LMV = (1-\alpha)(\frac{S_T}{xS_T})NC_{\{trigger\}} = (1-\alpha)N\frac{[M_{t_n}^T - L]^+ - [M_{t_n}^T - H]^+}{x(H-L)} \tag{6}$$

A higher x results in a lower conversion ratio and reduces the dilution losses of pre-existing shareholders when the conversion is triggered.

3.3 The CC-CoCos spread

To price these two features of CC-CoCos, we introduce the CC-CoCos spread (CC spread), the difference in coupon rate between a CC-CoCo and non-convertible bond, to capture CFV and LMV. We begin with a bank issuing a non-convertible bond at time t_n with a face value of *N* and a continuous coupon rate of *c* during term to maturity τ_t (or $T - t_n$). The bond price at maturity, time *T* is given by

$$P_T = N + \int_0^{T-t_n} cNds = N[1 + c(T - t_n)] = N[1 + c\tau_t] \tag{7}$$

Following Glasserman and Nouri (2012), we use a fixed risk-free rate, r_f, to discount all payoffs for pricing. Multiplying the discount factor with P_T in Eq. (7) gives us the following present value of the non-convertible bond:

$$P_{t_n} = Ne^{-r_f \tau_t} + \int_0^{\tau_t} cNe^{-r_f s} ds = N[e^{-r_f \tau_t}(1-\frac{c}{r_f}) + \frac{c}{r_f}]. \quad (8)$$

It is apparent from Eq. (8) that when the bond is issued at par (i.e., $P_t = N$), $c = r_f$.

Under the CCB framework, as the credit/GDP gap widens during credit expansion phases and exceeds the lower threshold, two contingent claims occur: CFV decreases and LMV is converted into common equity to increase capital buffers. We can therefore value CC-CoCos at time *T* from Eqs. (3), (6), and (7) as follows:

$$CC - CoCos_T = P_T - CFV + LMV = N + Nc\tau - NC_{\{trigger\}} + (1-\alpha)\frac{NC_{\{trigger\}}}{x} \quad (9)$$

For a numerical example, let $M_{t_n}^T$ = 6%, L = 2%, H =10%, α = 0% (zero write-down), and $C_{\{trigger\}}$ = 50%. Therefore, in addition to receiving the coupon payments of $Nc\tau$, CC-CoCos holders will receive a remaining face value of 0.5*N* and the market value of the bank share, $(\frac{0.5N}{x})$. If x = 1, CC-CoCos holders retain a total of *N*, half of which is in CC-CoCos and the other half in common equity. Therefore, there is no wealth transfer between CC-CoCos holders and pre-existing shareholders. However, if x > 1, CC-CoCos holders receive less than *N* and there is a transfer of wealth to shareholders. Conversely, if x < 1, CC-CoCos holders receive more than *N* and a transfer of wealth occurs from shareholders.

Adding the CC spread to the coupon rate of the non-convertible bonds for CC-CoCos, Eq. (9) can be rewritten as follows:

$$\begin{aligned} CC - CoCos_T &= N + Nc\tau - NC_{\{trigger\}} + (1-\alpha)\frac{NC_{\{trigger\}}}{x} \\ &= N + \int_0^{\tau_t} (c + CCspread)Nds - NC_{\{trigger\}} + (1-\alpha)\frac{NC_{\{trigger\}}}{x} \\ &= N[1 + (c + CCspread)\tau_t - NC_{\{trigger\}} + (1-\alpha)\frac{NC_{\{trigger\}}}{x} \end{aligned} \quad (10)$$

We can also express the present value of CC-CoCos in Eq. (8) as

$$
\begin{aligned}
CC-CoCos_{t_n} &= N[e^{-r_f\tau_t}(1-\frac{(c+CCspread)}{r_f})+\frac{(c+CCspread)}{r_f}] \\
&\quad -e^{(-r_f\tau_t)}E^Q[CFV]+e^{(-r_f\tau_t)}E^Q[LMV] \\
&= N[e^{-r_f\tau_t}(1-\frac{(c+CCspread)}{r_f})+\frac{(c+CCspread)}{r_f}] \\
&\quad +e^{(-r_f\tau_t)}(\frac{(1-\alpha)}{x}-1)E^Q[N\frac{[M_{t_n}^T-L]^+-[M_{t_n}^T-H]^+}{(H-L)}]
\end{aligned}
\tag{11}
$$

If CC-CoCos are issued at par, we can substitute c with r_f and $CC-CoCos_{t_n}$ with N in Eq. (11) and solve the CC spread as follows:

$$
\begin{aligned}
CCspread &= \frac{r_f}{N(1-e^{-r_f\tau_t})}e^{-r_f\tau_t}\{E^Q[CFV]-E^Q[LMV]\} \\
&= \frac{r_f}{N(1-e^{-r_f\tau_t})}e^{-r_f\tau_t}\{E^Q[NC_{\{trigger\}}]-E^Q[(1-\alpha)\frac{NC_{\{trigger\}}}{x}]\} \\
&= \frac{r_f[1-\frac{1-\alpha}{x}]}{(1-e^{-r_f\tau_t})}[e^{-r_f\tau_t}E^Q C_{\{trigger\}}] \\
&= \frac{r_f[1-\frac{1-\alpha}{x}]}{(1-e^{-r_f\tau_t})}[e^{-r_f\tau_t}E^Q\frac{[M_{t_n}^T-L]^+-[M_{t_n}^T-H]^+}{(H-L)}]
\end{aligned}
\tag{12}
$$

As expected, Eq. (12) shows that the CC spread positively varies with the principal write-down ratio (α) and the proportion of conversion price (x) as a percentage of S_T, which is the stock price at time *T*.

3.4 Closed-form solutions to the CC-CoCos spread

To obtain a closed-form solution to Eq. (12), we must determine the distribution of the credit/GDP gap, the underlying indicator of the CCB schemes. We therefore first develop a stochastic model to estimate the long-term trend of credit/GDP before establishing the distribution of the credit/GDP gap. We then incorporate the credit/GDP gap distribution into the CC spread in Eq. (12) using lookback options. We apply lookback options because their payoff, which is based on the underlying asset's optimal value over the life of the option, is equivalent to the full conversion of CC-CoCos into common equity over the term to maturity. The full conversion takes place when the credit/GDP gap equals or exceeds the higher threshold (H) prior to maturity. Therefore, it is consistent with the CCB schemes in which maximum capital buffers are required as the indicator (or trigger) equals or exceeds the higher threshold (H).

Drehmann et al. (2010) and Kauko (2012) show that private sector credit in credit/GDP displays non-stationarity and lacks mean reversion. In particular, private sector credit exposures in developed countries often exhibit an irregular growth, making it difficult to observe the upper growth limit. We therefore assume that private sector credit/GDP, X_{t_n}, follows the geometric Brownian motion (GBM) to determine the long-term trend of credit/GDP:

$$\frac{dX_{t_n}}{X_{t_n}} = \mu dt_n + \sigma dW_{t_n}^{p} \quad , \tag{13}$$

where μ and σ are instantaneous expected growth rate and volatility of credit/GDP respectively, and $dW_{t_n}^{p}$ is a Wiener process.[4] Since $\{\log(\frac{X_{t_i}}{X_{t_0}})\}_{i=0}^{T}$ can be decomposed into a trend and noise as follows:

$$\log(\frac{X_{t_i}}{X_{t_0}}) = b_{t_0}^{T}(t_i - t_0) + \varepsilon_{t_i} \qquad i = 0,1,...,T \, , \tag{14}$$

we can estimate $b_{t_0}^{T}$ in Eq. (14) using ordinary least square and obtain the following logarithm trend process:

$$\log(\frac{\hat{X}_{t_i}}{X_{t_0}}) = \hat{b}_{t_0}^{T}(t_i - t_0) \qquad i = 0,1,.....,T \, , \tag{15}$$

where $\hat{X}_T$ is the long-term trend of credit/GDP, X , from time t_0 to time *T*. To simplify the process of pricing the CC spread, we use the following three theorems for the three steps of pricing and provide the proofs in Appendix 1.

Theorem 1. If the credit/GDP, X_{t_n} , process defined in Eq. (13) is a lognormal distributed random variable, its long-term trend, $\hat{X}_T$, in Eq. (15) also conforms to the lognormal distribution.

According to Theorem 1, credit/GDP and its long-term trend follow the lognormal distribution. However, their difference, the credit/GDP gap, may not follow the same distribution as it can be a

[4] We follow Marathe and Ryan (2005) and run a GBM process fit test using US credit/GDP for 1961–2012. We find that the *p*-value of the log of credit/GDP from the Shapiro–Wilk W test is 0.423 and therefore, the null hypothesis of a lognormal distribution cannot be rejected. Furthermore, the *p*-value from the Chi-square test on two-way tables is 0.238.

negative lognormal distribution. Moreover, it can also be negatively or positively skewed. Borovkova et al. (2007) suggest that a "family" of log-normal distributions is well-suited to approximating the general gap or basket distributions. We use the moments-matching methods to approximate the lognormal family distributions of the credit/GDP gap in Theorem 2.

Theorem 2. For the two processes that follow GBM, the distribution of their differences can be first determined by solving a skewness coefficient, $\eta_{B(T)}$, and location parameters, τ, using the moments-matching method. The distribution is then classified into one of the four lognormal distribution family (regular, regular-shifted, negative, and negative-shifted) in Table 1 on the basis of $\eta_{B(T)}$ and τ.

After determining the distribution of the credit/GDP gap, we can estimate mean M_1 and volatility V under lognormal family distribution. We can then solve for $e^{-r_f \tau_t} E^Q[M_{t_n}^T - K]^+$ in Eq. (12) according to Theorem 3.

Theorem 3. $e^{-r_f \tau_t} E^Q[M_{t_n}^T - K]^+$ can be solved with mean M_1 and volatility V of the regular lognormal distribution using the lookback-gap option equations. The approximate solutions to the other three lognormal distribution families can be obtained after determining the parameters of scale *m*, shape *s*, and location τ in the first three moments of the distributions.

Using theorems 1, 2 and 3, we can obtain $e^{-r_f \tau_t} E^Q[M_{t_n}^T - K]^+$ and solve for Eq. (12). Since the pricing of the CC spread in Eq. (12) is related to initial M_1, it can be divided into three cases: M_1 < L, L < M < H, and M > H. There are four possible distributions of the credit/GDP gap with the three cases in each distribution. We summarize the approximate solutions of the 12 possible CC spreads in Table 2.

[Insert Table 2]

4. Empirical Results

4.1 Data and sample period

We demonstrate the effectiveness of CC-CoCos in converting into common equity or Tier 1 capital during periods of excessive credit growth and three–four years prior to a financial crisis. Following Drehmann et al. (2010), who report that the duration between two financial crises has a median of 15 year, we use a 15-year period to estimate the credit/GDP trend. We also follow Demiroglu et al. (2012) to determine the periods of credit cycles. We measure the tightness of credit using commercial and industrial (C&I) loan spreads, the difference between the C&I loan rate and the Fed fund rate. A higher (lower) C&I loan spread or a net tightening (loosening) of C&I loans indicates a period of credit contraction (expansion). We obtain data from the annual credit/GDP in US from the World Bank, Treasury bill rates from Federal Reserve Bank of St. Louis, and Commercial and Industrial (C&I) loans of large- and medium-sized firms from the Senior Loan Officer Opinion Survey.

Our sample period is 1990–2012.[5] It covers three credit contraction periods that begin from the early 1990:Q1 recession (Drehmann and Juselius (2014)), the collapse of the Internet bubble of 2001:Q1 (Lown and Morgan (2006)), and the recent financial crisis of 2007:Q3. Between these three contraction periods, there are two expansion periods that allow us to examine variations in the credit/GDP gap and the conversion of CC-CoCos. The annual data are converted into quarterly data to comply with the supervision frequency of the CCB regime.

4.2 Behavior of CC-CoCos conversion across credit cycles

[5] Although C&I loan data begin from 1967, there are missing observations for 1984–1990 and thus, our sample period begins from 1990.

First, the correlation coefficient between the credit/GDP gap, estimated using our stochastic long-term trend model and the C&I loan spread across the sample period, is −0.83. The negative correlation indicates that as the credit/GDP gap increases (decreases), the C&I loan spread narrows (widens) or credit conditions loosen (tighten). The variations in the credit/GDP gap estimates are therefore consistent with the different phases in the credit cycles.

Figure 3 shows the conversion rate of CC-CoCos over the sample period with the lower and higher thresholds at 2% and 10% respectively. The conversion begins during the credit loosening periods, that is, 3−4 years (e.g., 1996−2000 and 2003−2005) prior to a crisis, as indicated by the shaded areas. As the crises are approaching, more CC-CoCos are converted into common equity. The increase in Tier 1 capital from converting CC-CoCos therefore, meets the CCB requirements and prevents a higher cost of capital during or in the aftermath of the crises.

[Insert Figure 3]

4.3 Model parameter analyses

To further understand the pricing of the CC spread, we examine the three key groups of policy, loss absorption mechanism, and market parameters. We analyze the sensitivities of the CC spread to changes in these parameters using numerical examples. From our sample period, we obtain the averages of the inputs for the closed-form solutions in Table 1 to estimate the CC spread. They include credit/GDP = 162.91, credit/GDP growth rate (μ) = 2.63%, trend growth rates (θ) = 2.76%, and volatility of credit/GDP (σ) = 1.87%. The T-bill rates, as a proxy for risk free rates (r_f), for three-month, six-month, one-year, two-year, three-year, and five-year maturities are 3.53%, 3.68%, 3.83%, 4.17%, 4.40%, and 4.81%, respectively. We assume that the principal of CC-CoCos is 100, write-down ratio (α) is 0%, lower (L) and higher (H) trigger thresholds for conversion are 2% and 10% according to the CCB schemes, and conversion price C_p is 1.1 times the stock price at maturity since there is likely to be a share dilution after the conversion. From our sample, the skewness coefficient is $\eta_{B(T)} > 0$ and location parameter is $\tau < 0$. Therefore, the credit/GDP gap follows a regular shifted lognormal distribution, as reported in Table 1.

We can calculate the CC spreads from the closed-form solution on the basis of the regular shifted lognormal distribution, as shown in Table 2:

$M_1 < L$:

$$
\begin{aligned}
CCSpread = {} & \frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{C_{bs}(M_1-\tau, L-\tau) - C_{bs}(M_1-\tau, H-\tau) \\
& + (M_1-\tau)e^{-r_f\tau_t}[f(d_1(M_1-\tau, L-\tau))V - f(d_1(M_1-\tau, H-\tau))V] \\
& + (M_1-\tau)e^{-r_f\tau_t}N(d_1(M_1-\tau, L-\tau))\log(\frac{M_1-\tau}{L-\tau}) \\
& - (M_1-\tau)e^{-r_f\tau_t}N(d_1(M_1-\tau, H-\tau))\log(\frac{M_1-\tau}{H-\tau})\}
\end{aligned}
\tag{16}
$$

$L < M_1 < H$:

$$
\begin{aligned}
CCSpread = {} & \frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{C_{bs}(M_1-\tau, M_1-\tau) - C_{bs}(M_1-\tau, H-\tau) \\
& + (M_1-\tau)e^{-r_f\tau_t}[f(d_1(M_1-\tau, M_1-\tau))V - f(d_1(M_1-\tau, H-\tau))V] \\
& + e^{-r_f\tau_t}(M_1-L) - (M_1-\tau)e^{-r_f\tau_t}N(d_1(M_1-\tau, H-\tau))\log(\frac{M_1-\tau}{H-\tau})\}
\end{aligned}
\tag{17}
$$

$M_1 > L$:

$$
CCspread = \frac{r_f[1-\frac{(1-\alpha)}{x}]}{(e^{r_f\tau_t}-1)}
\tag{18}
$$

We first examine the effects of two policy parameters, trigger threshold and term of the historical trend of credit/GDP, on the CC spread. Figure 4(a) shows that a lower trigger threshold by the regulatory authority benefits the bank by increasing the chances of converting CoCos into common equity for capital buffers. Therefore, the CC spread is expected to be higher at the time of issuance. Figure 4(b) shows that when the historical trend is estimated for a longer duration under the regulatory policy, it tends to smooth out and become flatter. Therefore, the growth rates of the historical trend (θ) tend to be lower such that the gap between credit/GDP and its long-term trend is larger. As a result, the conversion rate increases and the CC spread is higher.

Figure 4 (a) and (b) also shows that the term structure of the CC spread exhibits a hump shape because the growth rates of credit/GDP are greater than those of historical trend, θ, in our sample period. From Eq. (11), it is apparent that as the credit/GDP gap increases over time, the CC spread will also be larger. However, as the conversion rate approaches the maximum of 100% (in year 3.25), the longer-term CC spread will decline because of the discount rate effect. The humped yield curve is consistent with the risky bond with the trigger conversion mechanism (default event) and write-down mechanism by Longstaff and Schwartz (1995).

[Insert Figure 4]

Second, we look at the effects of two parameters of the loss absorption mechanism, write-down ratio and conversion price. Figure 5(a) depicts the relationship between the CC spread and write-down ratio. As expected, a higher write-down ratio corresponds to a lower principal before conversion and therefore a higher CC spread is required by CC-CoCos holders. For example, at a write-down ratio of 60%, investors who receive the remaining 40% of principal require more that 25% of the Fed fund rate for the two-year CC-CoCos. Although the CC spread is higher at the time of issuance with a higher write-down ratio, the potential for stock dilution after the conversion is reduced.

Figure 5(b) shows that a higher conversion price, χ, (CC-CoCos investors pay more than the prevailing stock price for the conversion) is related to a lower share dilution effect and a higher

CC spread. It is noteworthy that when x is less than one (or 0.9, as shown in Figure 5(b)), the CC spread can become negative. In the current regime of the negative interest rates, banks can reduce the cost of CC-CoCos by offering negative coupon rates but with a lower conversion price, as depicted in the figure.

[Insert Figure 5]

Finally, Figure 6 (a) and (b) present the relationship between the two market parameters of our model, volatility of credit/GDP (σ) and growth rate of historical trend ($\hat{\theta}$), and the CC spread. Figure 6 (a) shows that as σ increases, the conversion rate and therefore, the CC spread, is expected to be higher. Figure 6 (b) illustrates that when $\hat{\theta}$ is lower, the credit/GDP gap widens, which in turn increases the conversion rate and CC spread. As $\hat{\theta}$ increases and the credit/GDP gap narrows, there is a lower probability of conversion or conversion rate. In such cases, the CC spread tends to monotonically increase with term to maturity. For example, Figure 6 (b) shows that the CC spread appears linear at $\hat{\theta}$ = 3.93% compared to the hump shape at $\hat{\theta}$ = 1.20%. The results are consistent with those of Sarig and Warga (1989) and Longstaff and Schwartz (1995), who find that the term structure of credit spreads monotonically increases for bonds with low trigger probability and is curvilinear for bonds with high trigger probability.

[Insert Figure 6]

5. Conclusions

In this study, we develop CC-CoCos, a financial innovation, that combines the basic characteristics of CoCs with the CCB framework of Basel III. The objective of doing so is twofold. First, it helps systematically boost capital buffers across banks by converting CC-CoCos into Tier 1 capital during periods of credit expansion. The increase in capital buffers corresponds with a build-up of systemic risk in the financial sector when credit/GDP exceeds well above its long-term trend. They also mitigate the negative signaling effect when the conversion takes place during good times. Second, by converting CC-CoCos into common equity using a macro-based trigger, it sidesteps several problems that plague the conventional CoCos, where the conversion is triggered by accounting or market value-based measures and occurs during the contraction phase. In particular, using credit/GDP as the trigger overcomes the opacity and manipulation problems related to accounting-based regulatory triggers and the death spiral effect and multiple pricing related to market-based triggers.

We show that depending on the dynamics of the credit/GDP gap and loss absorption requirement, the term structure of the CC spread can be positive or negative and hump shaped or monotonically increasing. Factors increasing the probability of conversion are likely to lead to a hump shape in the term structure of CC spread, while those lowering the probability are related to a normal yield curve, which is also the case with risk-free bonds. A conversion price that is greater than the stock price gives rise to a hump-shaped term structure; by contrast, a conversion price that is less than the stock price is associated with an inverted hump shape or negative CC spread.

The pricing of CC-CoCos is consistent with that of credit spreads for risky debt securities by Longstaff and Schwartz (1995), with the added features of a conversion trigger during the expansion phase of the credit cycles, that is, 3–5 years prior to a financial crisis. These features not only provide banks with additional capital prior to periods of credit contraction but also complement additional external capital that might be raised under the CCB framework. From a regulator's perspective, the designs of CC-CoCos conform to the macro-prudential policies of the Dodd-Frank Act and the recent BCBS proposals, in addition to the micro-prudential regulations on individual financial institutions. Furthermore, CC-CoCos can be viewed as a countervailing force toward banks' aggressive lending practices during periods of high credit growth by raising their cost of equity through conversion into common equity. It encourages banks to adjust their asset portfolios such as reducing high risk assets to satisfy their capital requirements. Therefore, they can behave as a stabilizer to slow down run-away credits.

A few caveats are in order here. First, the design of CC-CoCos is not intended to substitute conventional CoCos but rather to enhance the safety and soundness of the financial sector through the CCB framework. Therefore, CC-CoCos may work in conjunction with conventional CoCos at different phases of the credit cycles. Second, although credit/GDP has been recommended and found to be a reliable trigger, other macro-based triggers can also be effective in guiding the conversion of CC-CoCos. For example, triggers based on real estate indexes, stock market indexes, or CDS spread may prove to be effective measures of credit conditions in certain countries or market environments.

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Figure 1

Countercyclical capital buffer framework

This figure shows the CCB schemes as banks begin to build up capital buffers when the credit/GDP gap exceeds the lower threshold, which is 2% of the low threshold (L), and continues until the maximum of 2.5% of risk-weighted assets or 10% of the higher threshold (H).

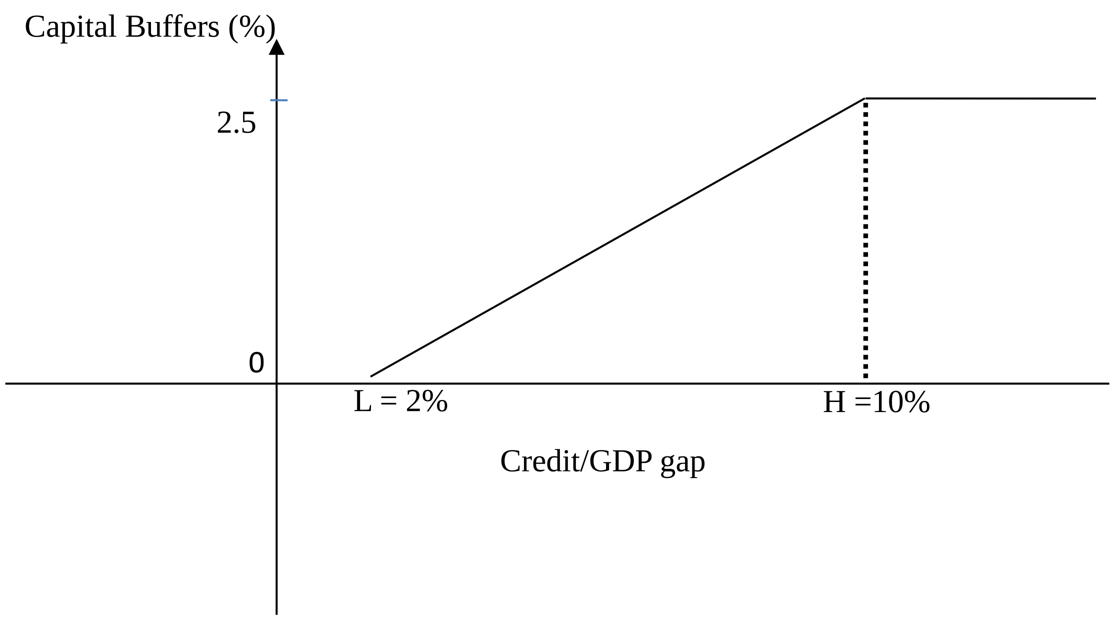

Figure 2

Countercyclical contingent capital instrument framework

This figure shows the structure of trigger and loss absorption mechanisms in the countercyclical contingent capital instrument (CC-CoCos) framework.

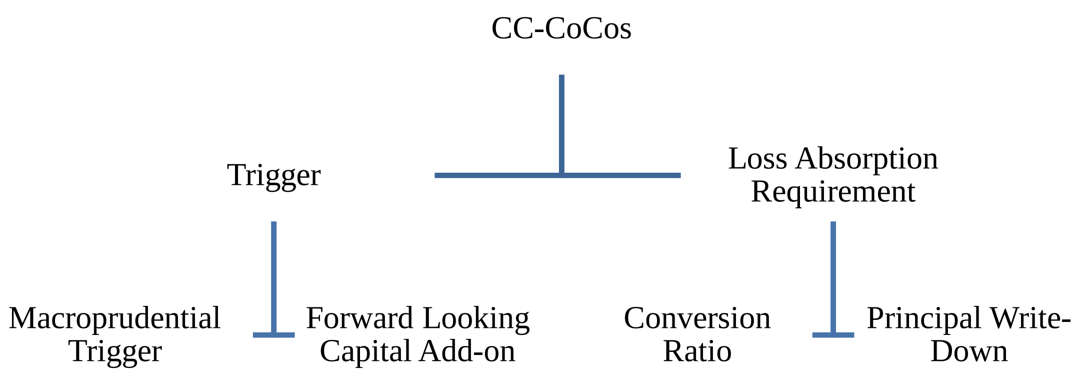

Figure 3

Credit conditions and conversion rates for 1990–2012

This figure depicts the variations in conversion rate across different credit conditions. The shaded areas indicate periods of credit contractions: 1990–1992, 2001–2003, and 2007–2009.

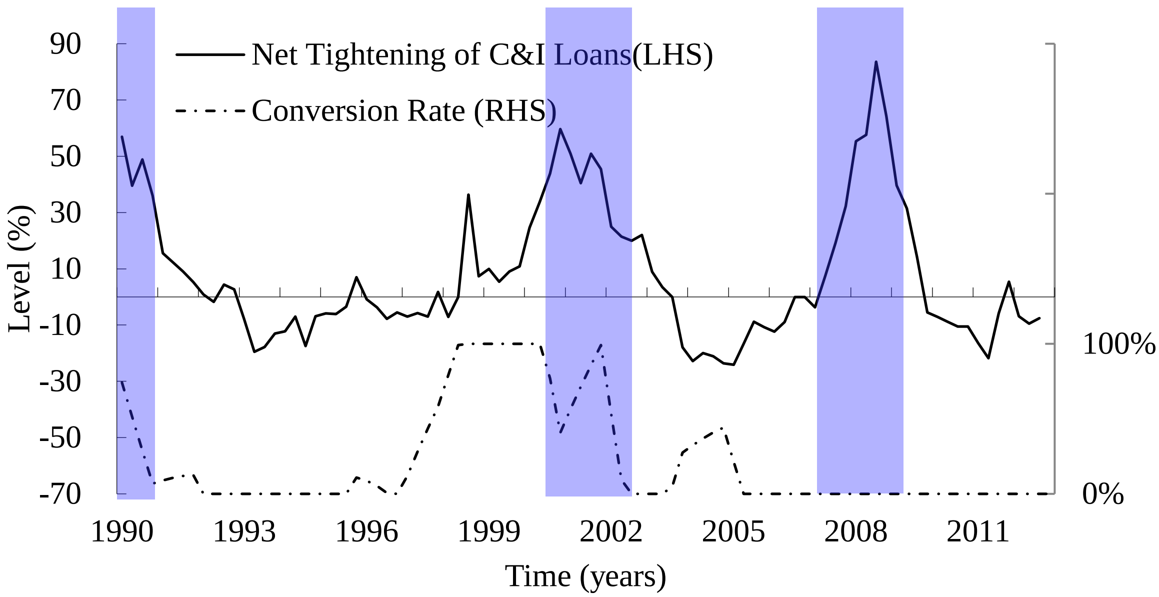

Figure 4

Policy parameters

Figure 4 (a) and 4 (b) depict the sensitivity of CC spread to changes in the trigger thresholds and changes in the time period of the long-term trend.

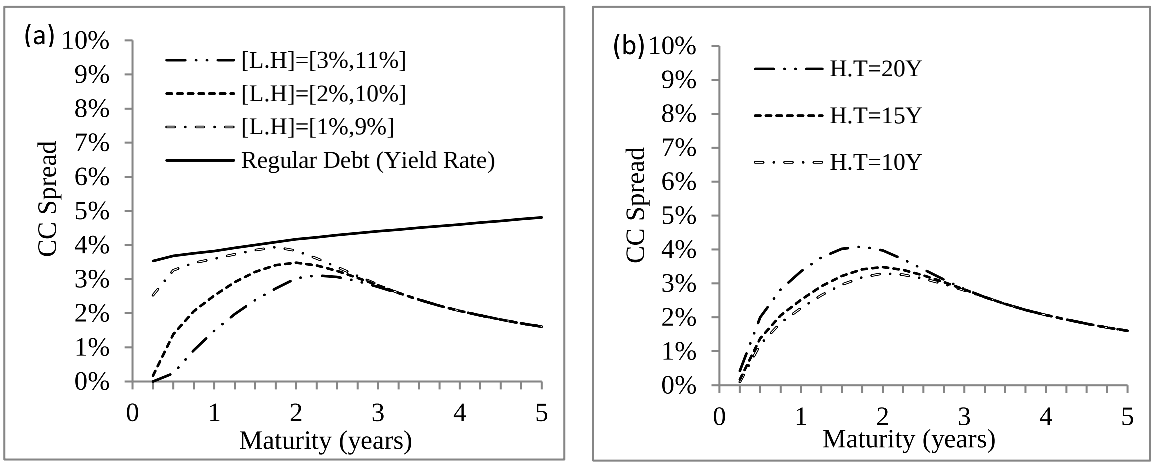

Figure 5

Loss absorption mechanism parameters

Figures 5 (a) and 5 (b) show the sensitivity of CC spread to changes in write-down ratio α and changes in conversion price *x*.

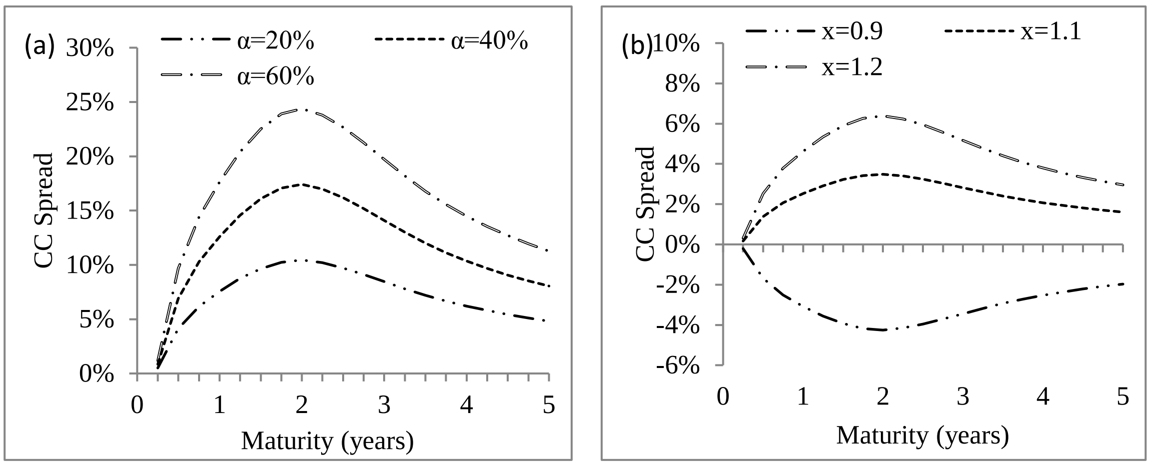

Figure 6

Market parameters

Figures 6 (a) and 6 (b) show the sensitivity of CC spread to changes in the volatility of credit-GDP and changes in the growth rate of historical trend θ.

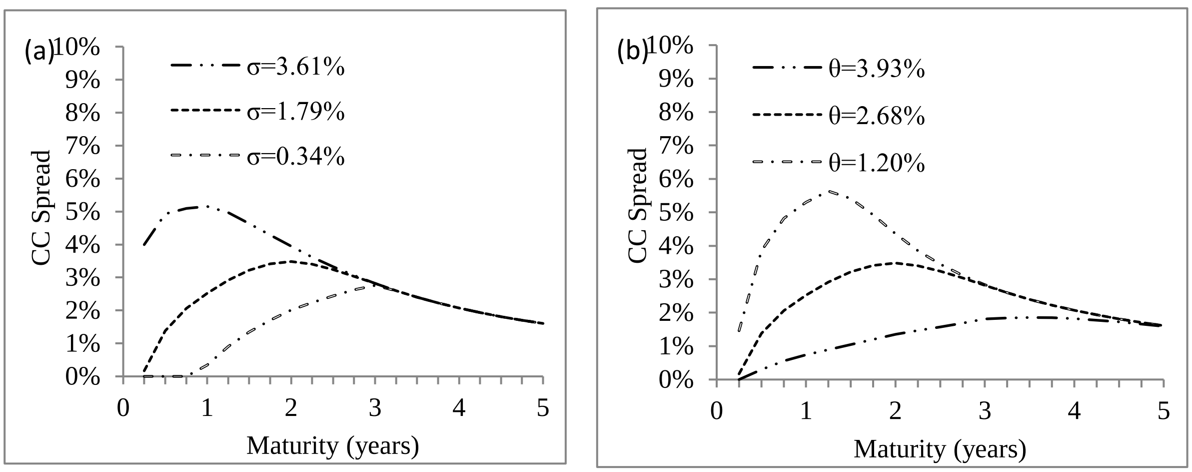

Table 1

Rules of approximating distribution of credit/GDP

This table presents the four types of lognormal distribution of credit/GDP according to the value of τ and $\eta_{B(T)}$: regular lognormal, regular shifted lognormal, negative lognormal, and negative shifted lognormal.

τ \ $\eta_{B(T)}$	> 0	< 0
> 0	Regular	Negative
< 0	Regular Shifted	Negative Shifted

Table 2

Closed-form solutions to CC spread

This table presents the closed-form solutions for the CC spread for each of the three cases in all four types of lognormal distribution of credit/GDP. The three cases are as follows: credit/GDP gap is less than the lower threshold ($M_1 < L$), credit/GDP gap is between the lower and higher thresholds ($L < M_1 < H$), and credit/GDP gap is greater than the higher threshold ($M_1 > L$). The four types of lognormal distribution are regular lognormal, regular shifted lognormal, negative lognormal, and negative shifted lognormal.

Distribution / Case	Regular Lognormal	Regular Shifted Lognormal
$M_1 < L$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{[C_{bs}(M_1,L)-C_{bs}(M_1,H)$ $+M_1e^{-r_f\tau_t}[f(d_1(M_1,L))V-f(d_1(M_1,H))V]$ $+M_1e^{-r_f\tau_t}N(d_1(M_1,L))\log(\frac{M_1}{L})$ $-M_1e^{-r_f\tau_t}N(d_1(M_1,L))\log(\frac{M_1}{H})\}$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{C_{bs}(M_1-\tau,L-\tau)-C_{bs}(M_1-\tau,H-\tau)$ $+(M_1-\tau)e^{-r_f\tau_t}[f(d_1(M_1-\tau,L-\tau))V-f(d_1(M_1-\tau,H-\tau))V]$ $+(M_1-\tau)e^{-r_f\tau_t}N(d_1(M_1-\tau,L-\tau))\log(\frac{M_1-\tau}{L-\tau})$ $-(M_1-\tau)e^{-r_f\tau_t}N(d_1(M_1-\tau,H-\tau))\log(\frac{M_1-\tau}{H-\tau})\}$

$L < M_1 < H$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{[C_{bs}(M_1,M_1)-C_{bs}(M_1,H)]$ $+M_1e^{-r_f\tau_t}[f(d_1(M_1,M_1))V-f(d_1(M_1,H))V]$ $+e^{-r_f\tau_t}(M_1-L)$ $-M_1e^{-r_f\tau_t}N(d_1(M_1,H))\log(\frac{M_1}{H})\}$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{C_{bs}(M_1-\tau,M_1-\tau)-C_{bs}(M_1-\tau,H-\tau)$ $+(M_1-\tau)e^{-r_f\tau_t}[f(d_1(M_1-\tau,M_1-\tau))V-f(d_1(M_1-\tau,H-\tau))V]$ $+e^{-r_f\tau_t}(M_1-L)$ $-(M_1-\tau)e^{-r_f\tau_t}N(d_1(M_1-\tau,H-\tau))\log(\frac{M_1-\tau}{H-\tau})\}$
$M_1 > H$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(e^{r_f\tau_t}-1)}$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(e^{r_f\tau_t}-1)}$

Distribution / Case	Negative Lognormal	Negative Shifted Lognormal
$M_1 < L$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{P_{bs}(-M_1,-L)-P_{bs}(-M_1,-H)$ $-M_1e^{-r_f\tau_t}[f(-d_1(-M_1,-L))V-f(-d_1(-M_1,-H))V$ $+M_1e^{-r_f\tau_t}N(-d_1(-M_1,-L))\log(\frac{-M_1}{-L})$ $-M_1e^{-r_f\tau_t}N(-d_1(-M_1,-H))\log(\frac{-M_1}{-H})$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{P_{bs}(-M_1-\tau,-L-\tau)-P_{bs}(-M_1-\tau,-H-\tau)$ $-(M_1+\tau)e^{-r_f\tau_t}(f(-d_1(-M_1-\tau,-L-\tau))V$ $-f(-d_1(-M_1-\tau,-H-\tau))V)$ $+(M_1+\tau)e^{-r_f\tau_t}N(-d_1(-M_1-\tau,-L-\tau))\log(\frac{-M_1-\tau}{-L-\tau})$ $-(M_1+\tau)e^{-r_f\tau_t}N(-d_1(-M_1-\tau,-H-\tau))\log(\frac{-M_1-\tau}{-H-\tau})$

$L < M_1 < H$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{P_{bs}(-M_1,-M_1)-P_{bs}(-M_1,-H)$ $-M_1e^{-r_f\tau_t}[f(-d_1(-M_1,-M_1))V-f(-d_1(-M_1,-H))V$ $+e^{-r_f\tau_t}(M_1-L)$ $-M_1e^{-r_f\tau_t}N(-d_1(-M_1,-H))\log(\frac{-M_1}{-H})$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(1-e^{-r_f\tau_t})(H-L)}\{P_{bs}(-M_1-\tau,-M_1-\tau)-P_{bs}(-M_1-\tau,-H-\tau)$ $-(M_1+\tau)e^{-r_f\tau_t}(f(-d_1(-M_1-\tau,-M_1-\tau))V$ $-f(-d_1(-M_1-\tau,-H-\tau))V)+e^{-r\tau_t}(M_1-K)$ $-(M_1+\tau)e^{-r_f\tau_t}N(-d_1(-M_1-\tau,-H-\tau))\log(\frac{-M_1-\tau}{-H-\tau})$
$M_1 > H$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(e^{r_f\tau_t}-1)}$	$\frac{r_f[1-\frac{(1-\alpha)}{x}]}{(e^{r_f\tau_t}-1)}$

Appendix 1

Proof of the Theorems

A. Proof of Theorem 1

When $T = t_0$, the intercept in Eq. (15) is zero. Hence, $\hat{b}_{t_0}^{T}$ with no intercept term can be obtained as

$$\hat{b}_{t_0}^{T} = \frac{\sum_{i=1}^{n+m}(t_i - t_0)(\log(\frac{X_{t_i}}{X_{t_0}}))}{\sum_{i=1}^{n+m}(t_i - t_0)^2} = \frac{6\sum_{i=1}^{n+m} t_i}{(n+m)(n+m+1)(2n+2m+1)\delta^2}\log(\frac{X_{t_i}}{X_{t_0}}),$$
$$= \sum_{i=1}^{n+m} \beta_i^{t_{n+m}} \log(\frac{X_{t_i}}{X_{t_0}})$$

(A1)

Where $\beta_i^{t_{n+m}} = \frac{6t_i}{(n+m)(n+m+1)(2n+2m+1)\delta^2}$, $\delta = t_i - t_{i-1} = \frac{(T - t_0)}{n+m}$, n is the number of periods from t_0 (time 0) to t_n (time when the option is issued), and m is the number of periods from t_n to T (i.e., t_{n+m}).

We can also relate $\hat{X}_T$ to X_{t_0} by using an exponent on both sides of Eq. (A1):

$$\hat{X}_T = X_{t_0} e^{\hat{b}_{t_0}^{T}(T - t_0)}.$$

(A2)

Let $\alpha_{t_i} = \sum_{j=i}^{n+m} \beta_j^{t_{n+m}}$; then, Eq. (A3) can be rewritten as

$$\hat{X}_T = X_{t_0}\exp((\alpha_{t_1}R_{t_1} + \alpha_{t_2}R_{t_2} + \cdots + \alpha_{t_{n+m}}R_{t_{n+m}})(T - t_0))$$
$$= X_{t_0}\exp((\hat{\theta} + \sum_{i=n+1}^{n+m}\alpha_{t_i}R_{t_i})(T - t_0)) \quad ,$$

(A3)

where $\hat{\theta} = \sum_{i=1}^{n}\alpha_{t_i}R_{t_i}$ is the historical log trend growth rate (hereinafter, historical trend growth rate) and $R_{t_i} = \log(\frac{X_{t_i}}{X_{t_{i-1}}})$ is the growth rate of one-period X_{t_i} distributed under the risk-neutral measure $N((r_f - \frac{1}{2}\sigma^2)\delta, \sigma^2\delta)$.

We use a 15-year period to estimate $\hat{X}_T$, which consists of $\hat{\theta}$ and R_{t_i}, in Eq. (A3). The choice of 15 years for the estimation period is based on Drehmann et al. (2010), who report that the duration between two crises ranges from 5 to 20 years, with a median of 15 years. If the term to maturity for CC-CoCos is one year, we use the period of 14 years prior to the issue of CC-CoCos to estimate $\hat{\theta}$ and one year to estimate R_{t_i}. If the term to maturity is two years, we use the period of 13 years prior to the issue of CC-CoCos to estimate $\hat{\theta}$ and two years to estimate R_{t_i}. It follows that the sum of $\hat{\theta}$ and R_{t_i} is 15 years for the estimation period.

Next, we examine the properties of α_{t_i} in Eq. (A3). They can be summarized in Lemma A.1 and Lemma A.2 with their proofs as follows:

Lemma A.1. $\alpha_{t_i} = \sum_{j=i}^{n+m}\beta_j^{t_{n+m}}$ (as defined earlier) has the following two properties:

$$Uc = \sum_{i=n+1}^{n+m} \alpha_{t_i} \delta = \delta \sum_{i=n+1}^{n+m} \sum_{j=i}^{m+n} \beta_j^{t_{n+m}} = \frac{m(m+1)(2m+3n+1)}{k(k+1)(2k+1)}, \tag{A4}$$

$$\begin{aligned} Sc &= (T - t_n) \sum_{i=n+1}^{n+m} \alpha^2{}_{t_i} \delta = m\delta^2 \sum_{i=n+1}^{n+m} \alpha^2{}_{t_i} \\ &= m\delta^2 \left(\sum_{i=1}^{n+m} \alpha^2{}_{t_i} - \sum_{i=1}^{m} \alpha^2{}_{t_i}\right) = m\delta^2 \left[\sum_{i=1}^{n+m} \left(\sum_{j=i}^{n+m} \beta_j^{t_{n+m}}\right)^2 - \sum_{i=1}^{n} \left(\sum_{j=i}^{n+m} \beta_j^{t_{n+m}}\right)^2\right] \\ &= \left\{\frac{6m[2k^2+2k+1]}{5k[2k^2+3k+1]} - \frac{9mn}{(2k+1)^2}\left[1 - \frac{(10k^2+10k-3n^2+2)(n^2-1)}{15k^2(k+1)^2}\right]\right\}, \end{aligned} \tag{A5}$$

where $n + m = k$.

Proof of Lemma A.1

The term $\beta_i^{t_{n+m}}$ in Eq. (A1), $\beta_i^{t_{n+m}} = \frac{6t_i}{(n+m)(n+m+1)(2n+2m+1)\delta^2}$, transforms Uc into the following:

$$Uc = \sum_{i=n+1}^{n+m} \alpha_{t_i} \delta = \delta \sum_{i=n+1}^{n+m} \sum_{j=i}^{m+n} \beta_j^{t_{n+m}} = \delta \sum_{i=1}^{m} i \beta_{n+i}^{t_{n+m}} = \frac{6\left(\sum_{i=1}^{m} ni + \sum_{i=1}^{m} i^2\right)}{(n+m)(n+m+1)(2n+2m+1)}, \tag{A6}$$

Uc can be expressed as the sum of two terms and simplified as follows:

$$\begin{aligned} \sum_{i=n+1}^{n+m} \alpha_{t_i} \delta &= \frac{3nm(m+1)}{(n+m)(n+m+1)(2n+2m+1)} + \frac{m(m+1)(2m+1)}{(n+m)(n+m+1)(2n+2m+1)} \\ &= \frac{m(m+1)(2m+3n+1)}{(n+m)(n+m+1)(2n+2m+1)}. \end{aligned} \tag{A7}$$

To show *Sc* (from *i* = n + 1 to n + m) in Eq. (A7) more clearly, we express it in the difference of two terms, *Sc*1 (from *i* = 1 to n + m) and *Sc*2 (from *i* = 1 to n), as follows:

$$Sc1 = m\delta^2 \sum_{i=1}^{n+m} (\sum_{j=i}^{k} \beta_j^{t_{n+m}})^2 = \frac{36m \sum_{i=1}^{k} (\sum_{j=i}^{k} j)^2}{k^2(k+1)^2(2k+1)^2} = \frac{6m(2k^2+2k+1)}{5k(k+1)(2k+1)} \quad \text{(A8)}$$

$$\begin{aligned} Sc2 &= m\delta^2 \sum_{i=1}^{n} (\sum_{j=i}^{k} \beta_j^{t_{n+m}})^2 = \frac{36m \sum_{i=1}^{n} (\sum_{j=i}^{k} j)^2}{k^2(k+1)^2(2k+1)^2} = \frac{36m \sum_{i=1}^{n} (\frac{k^2+k-i^2+i}{2})^2}{k^2(k+1)^2(2k+1)^2} \\ &= mn \times [\frac{9}{(2k+1)^2} - \frac{6(n^2-1)}{k(k+1)(2k+1)^2} + \frac{3(n^2-1)(3n^2-2)}{5k^2(k+1)^2(2k+1)^2}] \\ &= \frac{9mn}{(2k+1)^2}[1 - \frac{(10k^2+10k-3n^2+2)(n^2-1)}{15k^2(k+1)^2}]\} \end{aligned} \quad \text{(A9)}$$

Since Eq. (A3) can be rewritten as

$$\log \frac{\hat{X}_T}{\hat{X}_{t_n}} = (\hat{\theta} + \sum_{i=n+1}^{n+m} \alpha_{t_i} R_{t_i})(T - t_n) \quad \text{(A10)}$$

where by applying Lemma A.1, the long-term trend $\log \frac{\hat{X}_T}{\hat{X}_{t_n}}$ follows a standard normal distribution:

$$\begin{aligned} \log \frac{\hat{X}_T}{\hat{X}_{t_n}} &\sim N([\hat{\theta} + (r_f - \frac{1}{2}\sigma^2) \sum_{i=n+1}^{n+m} \alpha_{t_i} \delta](T - t_n), \sigma^2 (T - t_n)^2 \sum_{i=n+1}^{n+m} \alpha^2{}_{t_i} \delta) \\ &\sim N([\hat{\theta} + (r_f - \frac{1}{2}\sigma^2)\text{Uc}](T - t_n), \sigma^2 (T - t_n)\text{Sc}) \end{aligned} \quad \text{(A11)}$$

To differentiate from the volatility of credit/GDP, we define the variance in the long-term trend $v_{t_n}^{t_{n+m}}$ from Eq. (A11) as

$$v_{t_n}^{t_{n+m}} = \sigma^2 (T - t_n)^2 \sum_{i=n+1}^{n+m} \alpha^2{}_{t_i} \delta = \sigma^2 (T - t_n) \mathrm{Sc} \tag{A12}$$

It is apparent in Eq. (A12) that $v_{t_n}^{t_{n+m}}$ varies with n and m and differs from the variance in credit/GDP, $\sigma^2 (T - t_n)$, by a factor of Sc. From the distribution properties in Lemma A.1, we can now derive the correlation between credit/GDP and its logarithmic trend.

Lemma A.2. The correlation coefficient between credit/GDP and the long-term trend $\log \frac{\hat{X}_T}{\hat{X}_{t_n}}$ is

$$\hat{\rho}_{t_n}^{t_{n+m}} = \frac{\sigma^2 (T - t_n) \sum_{i=n+1}^{n+m} \alpha_{t_i} \delta}{\sqrt{v_{t_n}^{t_{n+m}}} \sigma \sqrt{(T - t_n)}} = \frac{\sigma \sqrt{(T - t_n)}}{\sqrt{v_{t_n}^{t_{n+m}}}} \times \mathrm{Uc} \tag{A13}$$

Proof of Lemma A.2

Malliaris and Brock (1982) show that the covariance between any two overlapping increments of a standard Gauss–Wiener process equals the smaller of the two time intervals:

$$Cov[W_{t_i}, W_{t_j}] = \min(t_i, t_j). \tag{A14}$$

From Eq. (A14), we obtain the covariance between logarithmic credit/GDP and its trend:

$$\begin{aligned} &Cov[Log(\frac{X_T}{X_{t_n}}), Log(\frac{\hat{X}_T}{\hat{X}_{t_n}}),] = Cov[R_{T-t_n}, (\hat{\theta} + \sum_{i=n+1}^{n+m} \alpha_{t_i} R_{t_i})(T - t_n)] \\ &= (T - t_n) \sum_{i=n+1}^{n+m} \alpha_{t_i} Cov[R_{T-t_n}, R_{t_i}] = (T - t_n) \sum_{i=n+1}^{n+m} \alpha_{t_i} \sigma^2 Cov[W_{T-t_n}, W_{t_i}] \\ &= (T - t_n) \sum_{i=n+1}^{n+m} \alpha_{t_i} \sigma^2 \min(T - t_n, \delta) = \sigma^2 (T - t_n) \delta \sum_{i=n+1}^{n+m} \alpha_{t_i} \end{aligned} \tag{A15}$$

Hence, $\hat{\rho}_{t_n}^{t_{n+m}} = \dfrac{\sigma^2 (T-t_n)\sum_{i=n+1}^{n+m} \alpha_{t_i} \delta}{\sqrt{v_{t_n}^{t_{n+m}}}\,\sigma\sqrt{(T-t_n)}} = \dfrac{\sigma\sqrt{(T-t_n)}}{\sqrt{v_{t_n}^{t_{n+m}}}} \times \mathrm{Uc}$

B. Proof of Theorem 2

According to our SLT approach, credit/GDP and its long-term trend both follow lognormal distributions. However, their difference or spread (i.e., credit/GDP gap) may not necessarily follow the same distribution because it may turn into a negative lognormal distribution. Therefore, we must assume that the credit/GDP gap distribution approximates one of four lognormal distributions: (1) regular log-normal (2) regular shifted log-normal (3) negative log-normal, and (4) negative- shifted log-normal.

In step 1, we use the moments-matching methods to approximate the lognormal distributions of the credit/GDP gap. From **Theorem 1**, the credit/GDP, $X_1(T)$, and its trend, $X_2(T)$, follow the correlated GBM with μ_i and σ_i. Following Vasicek (2003), who estimates the expected value of loans under the risk-neutral measure Q, we assume that the probability measures of credit $X_1(T)$ and its trend $X_2(T)$ are under Q measures with r_{mi} and σ_i. Based on this condition, the first three moments of the credit/GDP gap distribution, $B(T) = X_1(T) - X_2(T)$, at expiration date T are

$$E_{t_n}^{Q} B(T) = \sum_{i=1}^{2} a_i X_i(t) \exp[r_{mi}(T-t_n) + \frac{1}{2}\sigma_i^2 (T-t_n)], \tag{B1}$$

$$E_{t_n}^{Q}\left(B(T)\right)^2=\sum_{j=1}^{2}\sum_{i=1}^{2}a_i a_j X_i(t) X_j(t)\exp\left((r_{mi}+r_{mj})(T-t_n)+(\frac{\sigma_i^{\,2}+\sigma_j^{\,2}+2\sigma_{ij}}{2})(T-t_n)\right) \tag{B2}$$

$$\begin{aligned} E_{t_n}^{Q}\left(B(T)\right)^3 &= \sum_{k=1}^{2}\sum_{j=1}^{2}\sum_{i=1}^{2} a_i a_j a_k X_i(t) X_j(t) X_k(t) \\ &\times \exp\left[(r_{mi}+r_{mj}+r_{mk})(T-t_n)+[\frac{\sigma_i^{\,2}+\sigma_j^{\,2}+\sigma_k^{\,2}+2(\sigma_{ij}+\sigma_{ik}+\sigma_{jk})}{2}](T-t_n)\right] \end{aligned} \tag{B3}$$

where $(a_1, a_2) = (1,-1)$ is the weight vector. The skewness coefficient of the credit/GDP gap distribution $\eta_{B(T)}$ can be obtained from Eq. (B1) to (B3):

$$\eta_{B(T)} = \frac{E_{t_n}^{Q}[B(T) - E^{Q}[B(T)]^3}{s^3{}_{B(T)}}, \tag{B4}$$

where $s_{B(T)} = \sqrt{E_{t_n}^{Q}[B^2(T)] - (E_{t_n}^{Q}[B(T)])^2]}$.

As in Borovkova et al. (2007), the skewness $\eta_{B(T)}$ in Eq. (B4) can be used to determine whether the credit/GDP gap follows a regular or negative log-normal distribution. For example, if $\eta_{B(T)}$ is positive (negative), then the regular or regular shifted (negative or negative-shifted) log-normal distribution should apply.

Next, we construct the family of lognormal distributions using scale *m*, shape *s*, and location parameters *τ*. For example, when deriving the model parameters of a regular shifted log-normal distribution, the first three moments are

$$M_1 = \tau + \exp\left(m + \frac{1}{2}s^2\right), \tag{B5}$$

$$M_2 = \tau^2 + 2\tau \exp\left(m + \frac{1}{2}s^2\right) + \exp\left(2m + 2s^2\right), \quad \text{(B6)}$$

$$M_3 = \tau^3 + 3\tau^2 \exp\left(m + \frac{1}{2}s^2\right) + 3\tau \exp\left(2m + 2s^2\right) + \exp\left(3m + \frac{9}{2}s^2\right). \quad \text{(B7)}$$

Equating the credit/GDP gap moments in Eqs. (B1), (B2), and (B3) to the regular shifted log-normal distribution moments in Eqs. (B5), (B6), and (B7), we can solve for the model parameters (τ, m, s). If the credit/GDP gap follows a negative (or negative-shifted) log-normal distribution, we can solve the same non-linear equation system by replacing the first and third moments with $-M_1$ and $-M_3$. From the parameter τ, we can also determine whether the shift is needed. Table 2 summarizes the choices of lognormal distribution on the basis of τ and $\eta_{B(T)}$.

C. Proof of Theorem 3

We introduce the model parameters (m, s, τ) from futures contract to option pricing formula. We assume that the futures price F at time t over maturity T is F_t^T. If the spot price S follows regular lognormal distribution, its payoff under risk-neutral measure is (Brigo et al. (2003))

$$F_t^T = E_t^Q(S_T) = S_t e^{(r_f - q)\tau_t}, \quad \text{(C1)}$$

where r_f is the risk-free rate. Based on Musiela and Rutkowski (2005), the future option pricing formula with strike K and expiry date $\tau_t = (T - t_n)$ is

$$C_{bs}\left(F^T(t), K\right) = e^{-r_f \tau_t}\{F_t^T N(d_1\left(F_t^T, K\right)) - KN(d_2\left(F_t^T, K\right))\}, \quad \text{(C2)}$$

$$d_1\left(F_t^T,K\right)=\frac{\log(\frac{F_t^T}{K})+\frac{1}{2}V_{spot}{}^2\tau_t}{V_{spot}\sqrt{\tau_t}};\quad d_2\left(F_t^T,K\right)=d_1\left(F_t^T,K\right)-V_{spot}\sqrt{\tau_t}\ .$$

From Eq. (C2), the valuation of a future option is a special case of the Black–Scholes equation, where $r_f = q$ and the spot price is F_t^T. Similarly, if the lookback spot option valuation equation is known (see Wilmott (2006)), we can obtain the closed-form solution to the lookback future option for $r_f - q \rightarrow 0$ and $S_t = F_t^T$.

Note that the future option price in Eq. (C2) will be reduced to the regular Black–Scholes spot pricing formula if F_t^T is replaced by the first moment of spot price $S_t e^{(r_f-q)\tau_t}$ through Eq. (C1). Accordingly, Wu and Chen (2011) substitute the first moment of constant maturity swap for F_t^T in Eq. (C2) to obtain the approximated spread option price (see Brigo et al. (2003) and Borovkova et al. (2007) for a basket option).[6] Following these studies, we can derive the lookback-gap call option (LCK) by substituting the first moment of credit/GDP gap M_1 with $F_{t_s}^T$ and gap volatility V with V_{spot}, which are composed of parameters (m, s, τ) in the lookback-future option pricing formula.

When $F_t^T = M_1 \le K$,

$$LCK = C_{bs}\left(M_1,K\right)+M_1e^{-r_f\tau_t}f\left(d_1\left(M_1,K\right)\right)V+M_1e^{-r_f\tau_t}N\left(d_1\left(M_1,K\right)\right)\log\left(\frac{M_1}{K}\right) \qquad \text{(C3)}$$

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[6] They substitute the first moment of basket index for F_t^T into the future option pricing formula to approximate basket option price.

and when $F_t^T = M_1 > K$,

$$LCK = e^{-r_f \tau_t}\left(M_1 - K\right) + C_{bs}\left(M_1, M_1\right) + M_1 e^{-r_f \tau_t} f\left(d_1\left(M_1, M_1\right)\right)V \quad \text{(C4)}$$

Furthermore, LPK can be obtained as follows.

When $F_t^T = M_1 \le K$,

$$LPK = P_{bs}\left(M_1, K\right) + M_1 e^{-r_f \tau_t} f\left(-d_1\left(M_1, K\right)\right)V - M_1 e^{-r_f \tau_t} N\left(-d_1\left(M_1, K\right)\right)\log\left(\frac{M_1}{K}\right) \quad \text{(C5)}$$

and when $F_t^T = M_1 > K$,

$$LPK = e^{-r_f \tau_t}\left(K - M_1\right) + P_{bs}\left(M_1, M_1\right) + M_1 e^{-r_f \tau_t} f\left(-d_1\left(M_1, M_1\right)\right)V \quad \text{(C6)}$$

The pricing formulas of other lognormal distributions are determined using a variable transformation technique summarized below:

Case 1: Regular log-normal approximation

If the credit-/GDP gap, $B_1(T)$, follows the regular lognormal, where $\eta_{B(T)} > 0$ and $\tau > 0$, then the payoff of the options with strike price K is $(\max_{s\in[t_n,T]} B_1(s) - K)^+$. Let $\max_{s\in[t_n,T]} B_1(s) = M^{t^*}$ and $\min_{s\in[t_n,T]} B_1(s) = m^{t^*}$. The options can be divided into the following scenarios:

1. When $M_1 < K$, the credit/GDP gap is out-the-money at time t_n. From Eq. (C3), the option price with exercise price K is

$$LCK = e^{-r_f \tau_t} E^Q \left[\max\left(M^{t^*} - K, 0\right) \right]$$
$$= C_{bs}(M_1, K) + M_1 e^{-r_f \tau_t} f(d_1(M_1, K))V$$
$$+ M_1 e^{-r_f \tau_t} N(d_1(M_1, K)) \log(\frac{M_1}{K}), \tag{C7}$$

where

$$C_{bs}(M_1, K) = e^{-r_f \tau_t} \{M_1 N(d_1(M_1, K)) - KN(d_2(M_1, K))\},$$

$$d_1(M_1, K) = \frac{\log(\frac{M_1}{K}) + \frac{1}{2}V^2}{V}, \quad d_2(M_1, K) = d_1(M_1, K) - V,$$

$$V = \sqrt{\log(\frac{M_2}{M_1^2})\tau_t}.$$

2. When $M_1 \geq K$, the credit/GDP gap is in-the-money at time t_n. From Eq. (C4), the option price is

$$LCK = e^{-r_f \tau_t} E^Q \left[\max\left(M^{t^*} - K, 0\right) \right] = e^{-r_f \tau_t} (M_1 - K)$$
$$+ C_{bs}(M_1, M_1) + M_1 e^{-r_f \tau_t} f(d_1(M_1, M_1))V, \tag{C8}$$

where

$$C_{bs}(M_1, M_1) = e^{-r_f \tau_t} M_1 \{N(d_1(M_1, M_1)) - N(d_2(M_1, M_1))\},$$

$$d_1(M_1, M_1) = \frac{1}{2}V, \quad d_2(M_1, M_1) = -\frac{1}{2}V,$$

$$V = \sqrt{\log(\frac{M_2}{M_1^2})\tau_t}.$$

Case 2: Regular shifted log-normal approximation

If the credit-GDP gap, $B_2(T)$, follows a regular shift lognormal, where $\eta_{B(T)} > 0$ and $\tau <$ 0, it is also equals $B_1(T) + \tau$. Therefore, we can substitute $M_1 - \tau$ and $K - \tau$ for M_1 and K in the LCK options in Eqs. (C3) and (C4). The payoff of the option with strike price K is $(\max_{s\in[t_n,T]} B_2(s) - K)^+ = (\max_{s\in[t_n,T]} B_1(s) - (K-\tau))^+$.

1. If $B_2(T)$ is out-of-money at time t_n, the option price is

$$\begin{aligned} LCK &= e^{-r_f\tau_t} E^Q\left[\max\left(M^{t^*} - (K-\tau), 0\right)\right] \\ &= C_{bs-\tau}\left(M_1, K\right) + (M_1 - \tau)e^{-r_f\tau_t} f(d_{1-\tau}\left(M_1, K\right))V \\ &+ (M_1 - \tau)e^{-r_f\tau_t} N(d_{1-\tau}\left(M_1, K\right)) \log\left(\frac{M_1 - \tau}{K - \tau}\right), \end{aligned} \tag{C9}$$

where

$$C_{bs-\tau}\left(M_1, K\right) = e^{-r_f\tau_t}[(M_1 - \tau)N(d_{1-\tau}\left(M_1, K\right)) - (K - \tau)N(d_{2-\tau}\left(M_1, K\right))],$$

$$d_{1-\tau}\left(M_1, K\right) = \frac{\log\left(\frac{M_1 - \tau}{K - \tau}\right) + \frac{1}{2}V^2}{V}, \quad d_{2-\tau}\left(M_1, K\right) = d_{1-\tau}\left(M_1, K\right) - V,$$

$$V = \sqrt{\log\left(\frac{M_2 - 2\tau M_1 + \tau^2}{(M_1 - \tau)^2}\right)\tau_t}.$$

2. If $B_2(T)$ is in-the-money at time t_n, the option price is

$$\begin{aligned} LCK &= e^{-r_f\tau_t} E^Q\left[\max\left(M^{t^*} - (K-\tau), 0\right)\right] = e^{-r_f\tau_t}\left(M_1 - K\right) \\ &+ C_{bs-\tau}\left(M_1, M_1\right) + (M_1 - \tau)e^{-r_f\tau_t} f(d_{1-\tau}\left(M_1, M_1\right))V, \end{aligned} \tag{C10}$$

where

$$C_{bs-\tau}\left(M_1, M_1\right) = e^{-r_f\tau_t}(M_1 - \tau)[N(d_{1-\tau}\left(M_1, M_1\right)) - N(d_{2-\tau}\left(M_1, M_1\right))],$$

$$d_{1-\tau}(M_1, M_1) = \frac{1}{2}V, \quad d_2(M_1, M_1) = -\frac{1}{2}V,$$

$$V = \sqrt{\log\left(\frac{M_2 - 2\tau M_1 + \tau^2}{(M_1 - \tau)^2}\right)\tau_t}.$$

Case 3: Negative log-normal approximation

If the credit/GDP gap, $B_3(T)$, follows a negative lognormal, where $\eta_{B(T)} < 0$, $\tau > 0$ and $K < 0$, then we can also express $B_3(T)$ as $-B_1(T)$ and substitute $-M_1$ and $-K$ for M_1 and K in the LPK option in Eqs. (C5) and (C6). Therefore, the payoff of the option can be expressed as a put option:

$$(\max_{s\in[t_n,T]}(-B_1(T) - K)^+ = ((-K) - \min_{s\in[t_n,T]}(B_1(T))^+.$$

1. If $B_3(T)$ is out-of-money at time t_n, the option price is

$$\begin{aligned} LPK &= e^{-r_f\tau_t}E^Q\left[\max\left(-K - m^{t^*}, 0\right)\right] \\ &= P_{bs}(-M_1, -K) - M_1 e^{-r_f\tau_t} f(-d_1(-M_1, -K))V \\ &\quad + M_1 e^{-r_f\tau_t} N(-d_1(-M_1, -K))\log\left(\frac{-M_1}{-K}\right), \end{aligned} \tag{C11}$$

where

$$P_{bs}(-M_1, -K) = e^{-r_f\tau_t}\{-KN(-d_2(-M_1, -K)) + M_1 N(-d_1(-M_1, -K))\}$$

$$d_1(-M_1, -K) = \frac{\log\left(\frac{-M_1}{-K}\right) + \frac{1}{2}V^2}{V}, \quad d_2(-M_1, -K) = d_1(-M_1, -K) - V,$$

$$V = \sqrt{\log\left(\frac{M_2}{M_1^{\,2}}\right)\tau_t}.$$

2. If $B_3(T)$ is in-the-money at time t_n, the option price is

$$\begin{aligned} LPK &= e^{-r_f \tau_t} E^Q \left[\max\left(-K - m^{t^*}, 0\right)\right] = e^{-r_f \tau_t}(M_1 - K) \\ &+ P_{bs}\left(-M_1, -M_1\right) - M_1 e^{-r_f \tau_t} f(-d_1\left(-M_1, -M_1\right))V \end{aligned} \quad \text{(C12)}$$

where

$$P_{bs}\left(-M_1, -M_1\right) = e^{-r_f \tau_t} M_1\{-N(-d_2\left(-M_1, -M_1\right)) + N(-d_1\left(-M_1, -M_1\right))\},$$

$$d_1\left(-M_1, -M_1\right) = \frac{1}{2}V \; ; \; d_2\left(-M_1, -M_1\right) = -\frac{1}{2}V,$$

$$V = \sqrt{\log(\frac{M_2}{M_1^{\ 2}})\tau_t}.$$

Case 4: Negative shifted log-normal approximation

If the credit/GDP gap, $B_4(T)$, follows a negative shift lognormal, where $\eta_{B(T)} < 0$, $\tau < 0$ and $K + \tau < 0$, then $B_4(T)$ can be expressed in the form of $-(B_1(T) + \tau)$. Here as well, we can substitute $-M_1 - \tau$ and $-K - \tau$ for M_1 and K in Eqs. (C5) and (C6). The payoff of the option can be converted into a put option as follows:

$$(\max_{s \in [t_n, T]}(-B_1(T) - \tau) - K)^+ = (-(K + \tau) - \min_{s \in [t_n, T]}(B_1(T))^+.$$

1. If $B_4(T)$ is out-of-money at time t_n, the option price is

$$\begin{aligned} LPK &= e^{-r_f \tau_t} E^Q \left[\max\left(-(K + \tau) - m^{t^*}, 0\right)\right] = P_{bs-\tau}\left(-M_1, -K\right) \\ &- (M_1 + \tau)e^{-r_f \tau_t} f(-d_{1-\tau}\left(-M_1, -K\right))V \\ &+ (M_1 + \tau)e^{-r_f \tau_t} N(-d_{1-\tau}\left(-M_1, -K\right)) \log(\frac{-M_1 - \tau}{-K - \tau}) \end{aligned} \quad \text{(C13)}$$

where

$$P_{bs-\tau}\left(-M_1,-K\right)=$$
$$e^{-r_f\tau_t}\{(-K-\tau)N(-d_{2-\tau}\left(-M_1,-K\right))+(M_1+\tau)N(-d_{1-\tau}\left(-M_1,-K\right))\},$$

$$d_{1-\tau}\left(-M_1,-K\right)=\frac{\log(\frac{-M_1-\tau}{-K-\tau})+\frac{1}{2}V^2}{V},\quad d_{2-\tau}\left(-M_1,-K\right)=d_{1-\tau}\left(-M_1,-K\right)-V,$$

$$V=\sqrt{\log(\frac{M_2+2\tau M_1+\tau^2}{(M_1+\tau)^2})\tau_t}\,.$$

2. If $B_4(T)$ is in-the-money at time t_n, the option price is

$$LPK=e^{-r_f\tau_t}E\left[\max\left(-(K+\tau)-m^{t^*},0\right)\right]=e^{-r_f\tau_t}(M_1-K)$$
$$+P_{bs-\tau}\left(-M_1,-M_1\right)-(M_1+\tau)e^{-r_f\tau_t}f(-d_{1-\tau}\left(-M_1,-M_1\right))V, \qquad \text{(C14)}$$

where

$$P_{bs-\tau}\left(-M_1,-M_1\right)$$
$$=e^{-r_f\tau_t}(M_1+\tau)\{-N(-d_{2-\tau}\left(-M_1,-M_1\right))+N(-d_{1-\tau}\left(-M_1,-M_1\right))\},$$

$$d_{1-\tau}\left(-M_1,-M_1\right)=\frac{1}{2}V,\quad d_{2-\tau}\left(-M_1,-M_1\right)=-\frac{1}{2}V,$$

$$V=\sqrt{\log(\frac{M_2+2\tau M_1+\tau^2}{(M_1+\tau)^2})\tau_t}\,.$$