# Credit Default Swaps and Debt Overhang<sup>\*</sup>

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#### Abstract

We analyze the impact of credit default swaps (CDS) trading on firm investment and financing in a dynamic contingent claims model. Through the empty creditor channel, our model not only features a trade-off between debt capacity and costly bankruptcy but also shows that creditors' CDS protection allows a firm to capture a larger tax benefit at the expense of increasing agency cost. We quantify the agency cost of CDS as the loss in firm value induced by debt overhang. More precisely, CDS protection transfers firm future cash flows from shareholders to creditors, thereby discouraging the former from undertaking value-increasing investment projects. For firms with grim growth prospects, high business risk, or more tangible assets, the agency cost can be substantial. Moreover, we argue that debt overhang decreases with creditors' bargaining power, renegotiation frictions, and the debt's commitment to socially optimal credit insurance. The model yields a novel empirical implication that tests of the real impact of CDS trading need to account for times of debt issuance or refinancing.

Keywords: Credit Default Swaps, Debt Overhang, Investment, Empty Creditor, Credit Risk JEL Classification: G31, G33, G34

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## 1 Introduction

Credit default swaps (CDS) have become one of the most important financial innovations over the past two decades. Since the advent of the instrument, the CDS market has grown tremendously, primarily because it allows creditors to transfer credit risk out of their balance sheets and speculators to gamble on the prospects of the reference entities.<sup>1</sup> While there have been public debates about the desirability of the innovation and regulatory changes regarding the CDS market after the 2008 financial crisis, the impact of CDS trading on the real economic activities of firms has not been fully explored, particularly in dynamic environments. In this paper, we study the real impacts of credit derivative trading on current and future investment decisions and on the capital structure choices of CDS-referenced firms.

We introduce a competitive CDS market and dynamic investment opportunities in an otherwise standard Leland-type model. The key message of our paper is that CDS trading has both positive and negative sides. CDS trading allows lenders to extract more surplus from shareholders in debt renegotiation. This generates an *ex-ante* positive effect that relaxes financial constraints and allows the firm to exploit a larger tax shields during times of debt issuance. However, the wealth transfer from shareholders to creditors undermines the former's incentives to service debt and undertake investment. This results in an *ex-post* negative effect that forces the firm to forgo some positive net present value (NPV) projects and declare bankruptcy prematurely once the credit-protected debt is in place.

We construct a benchmark non-CDS firm that is free of the under-investment problem and identify CDS trading as a new source of debt overhang cost. We measure the loss in value, which we call the *agency cost of CDS*, using the relative difference in the values between the equitymaximizing and the value-maximizing CDS firms. Using the baseline calibration, we document that the magnitude of the agency cost of CDS is substantial relative to the debt overhang cost estimated in the existing literature. In fact, our comparative statics analysis shows that CDSinduced debt overhang destroys firm value for firms with grim growth prospects and high business risk or high liquidation value by 1 to 3 percent. We conclude that when evaluating the impact of CDS trading, it is important to account for the trade-off between increased debt capacity and increased debt overhang cost.

<sup>&</sup>lt;sup>1</sup>The CDS market peaked at USD 58,244 billion in a total notional amount in 2007. Since the financial crisis, the market has been shrinking. As of the first half of 2016, the market stood at USD 11,777 billion in notional amounts outstanding. Statistics are available at the Bank for International Settlements. (http://www.bis.org/statistics/derstats.htm, Table D10 OTC credit default swaps)

Our analysis builds on the well-known debt overhang problem analyzed by Myers (1977), and the empty creditor problem noted by Hu and Black (2008a,b), which was first formalized by Bolton and Oehmke (2011). The debt overhang problem refers to situations in which the equity holders have low incentives to undertake valuable investment because debt holders share the return of equity-financed investment in bankruptcy. The empty creditor problem arises when a firm's creditor, who has obtained insurance against bankruptcy, has no incentives to continue the firm efficiently and forces the debtor into inefficient liquidation. The CDS market provides the creditors with such an insurance instrument. In fact, the bargaining positions of credit-protected debt holders have been strengthened in out-of-court restructurings and their incentives to obtain credit insurance stem from the possibility of the equity to strategically default on its debt obligations.<sup>2</sup>

To study the interaction between debt overhang and empty creditors, we employ Leland's (1994) model of capital structure. On top of the standard trade-off between tax shields and bankruptcy costs, we endow the firm with dynamic investment opportunities. At any point in time, the equity holders can make an all-or-nothing decision regarding the growth of assets-in-place and have the option to liquidate the firm. Additionally, the debt is renegotiable: the equity holders have limited commitment to fulfilling the debt obligations and hence can default strategically and renegotiate the coupon with the creditors. As strategic debt service specifies a linear sharing rule, under our parametric restriction the levered firm is free of the debt overhang problem and always invests at the maximum level without the CDS market.

When the debt holders have access to a competitive CDS market that allows them to hedge against a firm's credit risk after the debt is in place, the CDS contracts purchased by the creditors increase their bargaining positions in private workouts. The reason is that once they reject the equity's proposal and exercise their liquidation rights, they receive the CDS payment from the protection sellers. The increase in the value of the outside option weakens the threat of liquidation imposed by the equity holders and allows the CDS-protected empty creditors to demand higher interest payments in renegotiation. CDS trading thus reduces the equity's incentives to renegotiate and default strategically. More importantly, as the credit insurance transfers wealth from the firm to the debt holders in private workouts, the equity holders must absorb more significant losses as the firm's fundamental deteriorates and hence accelerates the ex-post optimal bankruptcy time.

<sup>&</sup>lt;sup>2</sup>There is evidence that CDS trading affects corporate restructuring outcomes. Danis (2015) documents that firms with traded CDS have a lower bondholder participation rate in restructurings. Bedendo, Cathcart, and El-Jahel (2016) document increases in recovery prices in distress exchanges with empty creditors.

The endogenous default mechanism in Leland (1994) allows us to capture this logic conveniently.<sup>3</sup>

The CDS market then affects the firm's dynamic investment decisions through the empty creditor channel: debt overhang arises from the increased likelihood of bankruptcy with the inception of CDS trading. As the CDS accelerates the equity's bankruptcy time, the credit derivative endogenously shifts the distribution of investment benefits towards the debt holders and exacerbates the ex-post conflict of interests between the firm's owners and the lenders. Consequently, the equity stops investing early inefficiently as the firm's fundamental deteriorates and moves closer to the bankruptcy boundary. The resulting reduced asset growth rate post-introduction of the CDS market is thus the negative effect of credit insurance.

Notwithstanding the negative effect of CDS trading on the real side of the firm, the credit derivative commits the equity holders to renegotiate the debt contract less frequently. This exante commitment benefit expands the firm's debt capacity. Consistent with Bolton and Oehmke (2009), the increased borrowing ability allows financially constrained firms to finance a broader set of positive NPV projects initially. Additionally, as the debt holders are more willing to inject capital ex-ante, the firm can better exploit the tax shield and increases its value in general.

The dynamic nature of our model allows us to quantify the real effects of CDS trading. The optimal capital structure trades off the debt tax shields and costs of financial distress, including the amplified agency costs arising from debt overhang and accelerated bankruptcy through the empty creditor channel. With our baseline parameter constellations, the introduction of the CDS market raises the optimal market leverage from 37.04% to 58.15% and reduces the credit spread from 384 basis points to 117 basis points (bps).<sup>4</sup> The investment threshold and bankruptcy threshold increase from 0 to 3.83 and 2.77 respectively, resulting in a substantial non-investment region.

Consistent with the existing empirical literature, the increase in the optimal leverage and bankruptcy likelihood matches the evidence of Hirtle (2009), Saretto and Tookes (2013), and Subrahmanyam, Tang, and Wang (2014). The reduction in credit spread is consistent with the evidence of Ashcraft and Santos (2009) and Kim (2016). The real effects of CDS on dynamic investment are in line with those observed by Colonnello, Efing, and Zucchi (2016), Guest, Karapatsas, Petmezas,

 $<sup>^{3}</sup>$ We use "strategic default" and "renegotiation" interchangeably. We sometimes simply use "default" for formal bankruptcy.

<sup>&</sup>lt;sup>4</sup>The effect of CDS trading on debt market value is two-fold. First, Hackbarth, Hennessy, and Leland (2007) show that firms borrow up to the maximum coupon that triggers an immediate renegotiation. Any contractual coupon greater than the maximum is not credible and does not increase pledgeable income. In our model, this constraint is relaxed by creditors' CDS protection, resulting in an increase in the optimal coupon. Second, by deterring debt regeneration, it increases the market value of debt for a given coupon. These two sources and their interaction increase debt value significantly.

and Travlos (2017), and Batta and Yu (2017). Notably, Batta and Yu (2017) note that post-CDS introduction, the sampled firms have an average decline in asset growth by 2.1% and an overall reduction in net investment. However, when focusing on the CDS-introduction years, the evidence shows that both net investment and debt issuance increase in response to CDS trading. Our analysis thus provides CDS-driven debt overhang as one potential explanation for their observed dynamic investment pattern.

We quantify the *agency cost of CDS* as the percentage difference in the equity-maximizing and value-maximizing firm values. Using the baseline parameter values, we find that firm value would increase by approximately 0.4% if the equity holders would be able to commit to a firm value-maximizing investment policy after the debt is in place.<sup>5</sup> Interpreted as a new source of debt overhang, the 0.4% loss in firm value is economically non-trivial compared to the common estimates of debt overhang cost, which range from 1% to 5% (see our discussion following Table 1 for details). Moreover, we find that the estimated agency cost varies across firms with different characteristics. Focusing on firms with poor investment opportunities, we document that the agency cost of CDS can increase by a range of 1% to 3%, depending on other firm's characteristics including cash-flow volatility and bankruptcy cost.

We also consider an alternative measure that identifies the debt overhang cost as the reduction in asset value due to CDS trading. To that end, we choose an otherwise identical firm but with no access to CDS trading (a non-CDS firm) as a benchmark. Under the baseline parameters, the reduction is approximately 2.0% when we scale the reduction with the value of the non-CDS firm and is approximately 2.2% when we take the unlevered asset of the non-CDS firms as the scale. Both quantities show that the asset reduction is non-trivial and possibly massive in dollar terms for large corporations.

The baseline model allows us to provide a few more results. First, debt overhang decreases with the bargaining power of the debt holders. Intuitively, as the key benefit of CDS protection to the debt is the strength it provides in negotiation, so the increase in the debt's bargaining power substitutes for the use of a CDS contract. In the extreme case when the debt holders can make a take-it-or-leave-it offer to the equity, the debt effectively becomes the residual claimant; to minimize

<sup>&</sup>lt;sup>5</sup>In dollar terms, the agency cost amounts to approximately \$400,000 for a firm with \$100 million total assets. It is worth noting that in our model firm value comes from the unlevered assets, growth options, and financial leverage. For a firm with \$100 million in total assets (book value), \$400,000 is perhaps a lower bound of the estimated dollar amount of agency cost. The agency cost (in percentage points) is even greater if we scale it by the value of unlevered assets, a measure that corresponds to the total assets (book value) that is widely used in the empirical studies as a proxy for firm size. Put differently, a firm with \$100 million in total assets (book value) and an 1.6 book-to-market ratio has a market value of \$160 million, implying an agency cost of \$640,000.

the liquidation costs, the debt holders forgo the hedging opportunities. Thus, our analysis implies a minimal overhang in this case.

Second, debt overhang decreases with renegotiation frictions. The reason is that when the renegotiation cost is high, the incentive for the equity to restructure its debt is low. This reduces the creditors' use of CDS protection, the likelihood of bankruptcy, and hence the overhang cost.

Finally, the debt holders hedge excessively against credit risk. When the creditors can freely choose their CDS positions, they trade off the commitment benefits of reducing strategic default (debt renegotiation) and the costs of CDS premium from increasing bankruptcy. Nevertheless, the privately optimal level of credit protection is inefficient: the creditors over-insure against bankruptcy because they do not fully internalize the equity's loss of the option value to rene-gotiate out-of-court and the bankruptcy costs. Compared to the social optimum, the excessive credit protection chosen by the empty creditors exaggerates debt overhang.

Our paper contributes to a growing literature that examines the impact of CDS trading from a corporate finance perspective.<sup>6</sup> First, we provide a structural model with CDS being non-redundant securities. On the theoretical side, we embed the empty creditor problem in Bolton and Oehmke (2011) into the dynamic contingent claims model with renegotiable debt and investment opportunities.<sup>7</sup> We are among the first to study CDS as a non-redundant security in the structural credit risk model. Kim (2016) introduces the CDSs in a dynamic model with debt-for-equity swaps. However, he only uses the model to motivate the test hypotheses for his empirical investigation without analyzing the dynamic path of investment.

Second, we identify debt overhang as a new source of inefficiency through the empty creditor channel. The most closely related paper to us is Bolton and Oehmke (2011). They analyze a singleshot investment problem and derive the ex-ante commitment benefits of CDS trading in reducing strategic default. Using a continuous-time setting in the spirit of Leland (1994) and Diamond and He (2014), we extend the empty creditor analysis by considering not only the tax benefits of debt but also the firm's dynamic investment decisions. We provide a few novel implications. First, we show that lenders' CDS protection can increase firm value even for unconstrained firms, whereas Bolton and Oehmke's (2011) results are primarily concentrated among financially constrained firms. Second, we establish an intertemporal relation between CDS protection and investment. As in their paper, CDS protection allows the creditors to extract more future cash flows in debt renegotiations

<sup>&</sup>lt;sup>6</sup>Augustin, Subrahmanyam, Tang, and Wang (2016) provide an excellent summary.

<sup>&</sup>lt;sup>7</sup>See Mella-Barral and Perraudin (1997), Fan and Sundareasan (2000), Hennessy (2004), Hackbarth, Hennesy, and Leland (2007), Davydenko and Strebulaev (2007), and Diamond and He (2014).

and mitigates under-investment ex-ante. However, our model with dynamics investment reveals that the empty creditor effect amplifies under-investment ex-post. Third, lenders always overinsure against credit risk in our model.

In a contemporaneous and related work, Danis and Gamba (2018) extend Bolton and Oehmke (2011) into a dynamic setting with costly equity financing and one-period debt contracts. Their calibration shows that investment, leverage, and firm value increase with the inception of CDS trading. In their model, simultaneous investment and financing decisions imply that investment maximizes total firm value. Our paper has a different focus. Instead, we solve for the dynamic investment decisions that are made after initial debt issuance and CDS protection. This different timing of corporate policies allows us to derive an intertemporal relation between CDS protection, debt issuance, and corporate investment. To the best of our knowledge, we are the first to show that firms facing CDS-protected creditors will pre-maturely abandon value-increasing investment projects.<sup>8</sup>

In addition, our model sheds light on theoretical works that study CDS markets. Oehmke and Zawadowski (2015) analyze the effect of CDS introduction on bond prices and CDS-Bond basis in an equilibrium model. Parlour and Winton (2013) analyze the trade-off between loan sales and CDS regarding credit risk transfers and monitoring incentives in a banking model. Campello and Matta (2016) argue that CDS over-insurance is pro-cyclical and CDS trading leads to a more significant increase in debt capacity during economic booms. Fostel and Geanakoplos (2016) show that uncovered CDS positions may lead to under-investment. In contrast, covered CDS positions that distort the debtor-creditor relationship drives our debt-overhang result.

Lastly, we propose several new testable hypotheses regarding the CDS firms' dynamic paths, which call for the CDS firms' debt issuance times to be distinguished from their true dynamics. Our dynamic model suggests that CDS firms capture the ex-ante positive effect of debt capacity, and, therefore, have increased debt-financed investment at any debt issuance or refinancing times.

<sup>&</sup>lt;sup>8</sup>Our paper is also different from Danis and Gamba (2018) in several other dimensions. First, unlike Danis and Gamba (2017) who use a reduced form modeling for the debt-equity choice, we endogenize the comparative advantage and cost of debt over equity by incorporating tax benefits and agency cost (underinvestment cost) of debt. Second, and related to the first, we derive the equilibrium cost of debt (credit spread) under the optimal capital structure. Our numerical analysis reveals that firms borrowing from CDS-protected creditors have a lower cost of debt despite higher (optimal) financial leverage. Third, we show that the overall value effect of CDS protection could be negative. To be precise, CDS protection may decrease firm value for firms with poor investment opportunity and risky cash flow or having assets with low liquidation costs. Fourth, we find that creditors over-insure against default risk compared to a firm value maximizer. As both debt overhang and bankruptcy increases in CDS protection (see Section 3.2.2), this result yields a novel policy implication for financial industry regulators. Finally, our continuous-time model permits a closed-form characterization of the renegotiation threshold. We also obtain an analytical result that CDS protection accelerates underinvestment and default (Proposition 1).

Once the debt is in place, recapitalization costs prevent a CDS firm from tapping the debt market often, and during those times, the ex-post negative effect of investment emerges. The distinction between debt issuance (refinancing) times and the true dynamics is reminiscent of Strebulaev (2007). Along these lines, our framework provides a consistent explanation of the estimations reported by Colonnello, Efing, and Zucchi (2016), Batta and Yu (2017), and Guest, Karampatsas, Petmezas, and Travlos (2017). For financial implications, our work provides a more complete theoretical foundation to several empirical findings that relate CDS and corporate finance, such as those of Ashcraft and Santos (2009), Hirtle (2009), Saretto and Tookes (2013), Subrahmanyam, Tang, and Wang (2014), and Kim (2016).

### 2 The Model

**Technology.** Consider a firm with assets that generate pre-tax cash flows at rate  $\delta_t$ . The cash-flow process  $\{\delta_t : t \ge 0\}$  evolves as a geometric Brownian motion under the risk-neutral measure

$$\frac{d\delta_t}{\delta_t} = (\mu + i_t) \, dt + \sigma dZ_t.$$

The baseline growth rate is  $\mu$ , the volatility  $\sigma$  is a positive constant, and  $\{Z_t : t \ge 0\}$  is a standard Brownian motion.<sup>9</sup> Endogenous investment  $i_t \in \{0, i\}$  affects the asset growth rate. The investment cost is given by  $\phi i_t \delta_t$  because the asset growth scales with  $\delta_t$ . Following Myers (1977), the equity, or the firm's manager who acts in the best interest of the shareholders, controls the investment decisions. Investment costs are equity-financed.

As in other Leland-type models, we let  $\tau \in [0, 1]$  be the corporate tax rate and  $\alpha \in [0, 1]$  be a proportional bankruptcy cost.<sup>10</sup> If an unlevered firm always invests, its asset value is

$$\mathbb{E}_t \left[ \int_t^\infty e^{-r(s-t)} \left( (1-\tau)\delta_s - \phi i_s \delta_s \right) ds \right] = \frac{(1-\tau)(1-\tilde{\phi}i)}{r-(\mu+i)} \delta_t.$$
(1)

In contrast, if the firm never invests, its asset value is given by  $\frac{1-\tau}{r-\mu}\delta_t$ . In (1), we define  $\tilde{\phi} \equiv \frac{\phi}{1-\tau}$  and assume  $r > \mu + i$  for convergence. Additionally, for notational convenience, we define  $U_i \equiv \frac{1-\tilde{\phi}i}{r-(\mu+i)}$ ,

<sup>&</sup>lt;sup>9</sup>All stochastic processes are defined on a complete probability space  $(\Omega, \mathcal{F}, P)$  with the filtration  $\mathbb{F} = \{\mathcal{F}_t : t \ge 0\}$  satisfying the usual conditions. Diamond and He (2014) use this cash-flow specification to examine the implication of debt maturity on overhang.

<sup>&</sup>lt;sup>10</sup>As in Myers (1977), corporate taxes are not crucial for our key results on debt overhang: Proposition 1 and 5 hold with  $\tau = 0$ . However, the tax shield provides incentives for the firm to use debt financing, so taxes are important for optimal leverages and credit risks in the quantitative analysis.

 $U_0 \equiv \frac{1}{r-\mu}$ , and assume  $\Pi \equiv U_i - U_0 > 0$ . Hence,  $\Pi > 0$ , or equivalently  $U_0 > \tilde{\phi}$ , captures a positive marginal value of investment; and (1) provides the unlevered firm value. In the quantitative analysis, we use the parameter  $\Pi/U_0$  to measure the profitability of investment opportunities, which provides a proxy for growth options.

The liquidation value of the asset is  $L\delta_t$ , where  $L \equiv (1-\alpha)\frac{1-\tau}{r-\mu}$ . That is, once the firm liquidates, it loses its investment opportunities or its existing manager who has superior skills in investment. We maintain the following parametric restriction throughout the paper.

## Assumption 1. $U_i - \tilde{\phi} > L$ .

While  $\Pi$  captures the investment value from the *firm*'s perspective,  $U_i - L - \tilde{\phi}$  measures the net investment value accruing to the *shareholders* of a levered firm given the present value of interest payment per unit of cash flows is L. To see this, consider an increase in the investment from 0 to a level such that  $i_t \delta_t dt = 1$  over a small time interval (t, t + dt). The additional unit of fundamental generates a present value of  $U_i$  and it costs  $\tilde{\phi}$ . Moreover, suppose that the extra interest is L, which will occur in a benchmark without the credit default swaps (CDS) market (see Section 3.3.1), then Assumption 1 ensures that the equity holders have sufficient incentives to invest under the given debt service.

**Financing.** The firm has access to a frictionless equity market and a debt market. At time 0, the firm borrows from outside investors by issuing a perpetual debt contract that promises a contractual coupon at rate  $c_B$ . When the equity holders declare bankruptcy, the debt holders have absolute priority in liquidation and continue to run the firm as an ongoing entity or sell the firm outright. Hence, the debt holders obtain the liquidation value  $L\delta_t$ .<sup>11</sup>

However, the equity holders can initiate a private workout to renegotiate the interest payment at any point in time because of their limited commitment to fulfilling the debt obligations. We follow the approach by Mella-Barral and Parraudin (1997) and Hackbarth et al. (2007) in modeling renegotiation (strategic default). Let  $s(\delta)$  be the debt service flow function. In renegotiation, the equity holders make a take-it-or-leave-it offer  $s(\delta)$  to the debt holders.<sup>12</sup> The incentive for the debt

<sup>&</sup>lt;sup>11</sup>Following other Leland-type models, we assume the new firm's owner does not relever.

<sup>&</sup>lt;sup>12</sup>Fan and Sundarasean (2000) and Sundarasean and Wang (2007) model renegotiation as a Nash bargaining game. In contrast, the equity holders make a take-it-or-leave-it offer and they have all the bargaining power in our model. In Section 5.1, we consider take-it-of-leaving-it offers made by debt holders.

holders to accept the offer stems from the threat of liquidation. The threat is credible because the debt bears part of the bankruptcy cost in a negotiation breakdown. Since the debt holders obtain their reservation value  $R(\delta)$  upon exercising the liquidation right, they accept the proposal  $s(\delta)$  if it delivers at least the reservation value.<sup>13</sup>

The CDS market. The novel element in our model is that the creditors may enter into a single-name CDS contract that references the firm's debt. As protection buyers, the creditors can (partially) transfer their losses in bankruptcy to CDS sellers. Specifically, we model the CDS market as a competitive market with risk-neutral protection buyers and sellers. Right after the debt issuance and before the firm operates, the debt holders can decide their CDS position. We denote the position as  $\theta \in [0, \infty)$ , which represents a lump-sum payment made by the protection seller to the buyer in a credit event. In exchange, the protection buyer pays a CDS premium at rate p to the seller. Here, only bankruptcy qualifies as a credit event, and a successful renegotiation does not trigger the credit protection. We make this assumption because the International Swaps and Derivatives Association (ISDA) no longer recognizes debt restructuring as a credit event since Spring 2009. For tractability, we assume that CDS contracts are perpetual to match the debt's maturity.

In a competitive market, CDS contracts are fairly priced. Given a position  $\theta$ , the seller sets a CDS premium p to break even:

$$\mathbb{E}\left[\int_0^{\tau_d(\theta)} e^{-rt}(-p)dt + e^{-r\tau_d(\theta)}\theta\right] = 0.$$
 (2)

Here,  $\tau_d(\theta)$  is the bankruptcy time chosen by the equity given the debt's credit protection. Therefore, the novelty of the pricing equation (2) is that it incorporates the interaction between the equity's ex-post decisions and CDS trading and highlights the role of endogenous decisions as an essential mechanism in determining the CDS premium with renegotiable debt. The competitive market pricing mechanism is in sharp contrast to reduced-form credit risk models, which typically price CDS contracts based on the exogenously specified bankruptcy intensity and liquidity process.

<sup>&</sup>lt;sup>13</sup>We follow previous studies and restrict attention to debt service functions that are piecewise right continuous in  $\delta$  and  $s(\delta) < c_B \Rightarrow b(\delta) \ge R(\delta)$ , where  $b(\delta)$  is the debt's payoffs defined in Section 3.2.1.

## 3 Model Analysis

In this section, we first fix the capital structure and derive the closed-form valuation formulas for CDS contracts and corporate securities. In the process, we characterize the equity's optimal decisions after the debt is in place, the key debt-overhang result, and the debt's optimal hedging strategy given the anticipation of the equity's ex-post optimal policies. We then analyze two benchmarks: non-CDS firms and CDS firms with a commitment to an efficient investment policy. The benchmarks allow us to characterize the debt overhang cost induced by CDS trading. We end the section by deriving the optimal capital structure.

We characterize the optimal policies by three endogenous thresholds. We denote  $\delta_i$  as an investment threshold,  $\delta_n$  as a renegotiation threshold, and  $\delta_d$  as a bankruptcy threshold. Moreover, because under-investment may occur when the firm's fundamental deteriorates, we assume the equity's investment policy takes the following form:

$$i(\delta) = \begin{cases} i, & \text{if } \delta \ge \delta_i; \\ 0, & \text{if } \delta < \delta_i. \end{cases}$$

Throughout the paper, we assume  $\delta_n > \delta_i$ .<sup>14</sup> In what follows,  $z_1 < 0$  is the negative root, and  $a_1 > 1$  is the positive root of the fundamental quadratic  $Q_1(x) = \frac{1}{2}\sigma^2 x^2 + (\mu + i - \frac{1}{2}\sigma^2) x - r = 0$ ;  $z_0 < 0$  is the negative root; and  $a_0 > 1$  is the positive root of the fundamental quadratic  $Q_0(x) = \frac{1}{2}\sigma^2 x^2 + (\mu - \frac{1}{2}\sigma^2) x - r = 0$ . Note that as i > 0,  $0 > z_0 > z_1$  and  $a_0 > a_1 > 1$ .

### 3.1 Valuation of Credit Default Swaps

The value of a CDS contract contains its protection leg and its premium leg. The protection leg involves no cash changing hands until formal bankruptcy where the CDS seller makes a lump-sum payment  $\theta$  to its buyer. Its value is  $C(\delta) = \mathbb{E}^{\delta} \left[ e^{-r\tau_d(\theta)} \theta \right]$  and has a closed-form solution:

$$C(\delta) = \begin{cases} \theta P_d^i(\delta), & \text{if } \delta \ge \delta_i; \\ \theta P_d^0(\delta), & \text{if } \delta_d < \delta < \delta_i \end{cases}$$

where  $P_d^i(\delta)$  is the present value of a contingent claim that pays one dollar at bankruptcy for  $\delta$ in the investment region, and  $P_d^0(\delta)$  is for  $\delta$  in the non-investment region. Intuitively, the quan-

<sup>&</sup>lt;sup>14</sup>We make this assumption because for all reasonable parameter values we work with, we find that  $\delta_n > \delta_i$ .

tities capture the probability of bankruptcy at state  $\delta$ . The appendix provides their closed-form expressions.

The value of the premium leg is  $P(\delta) = \mathbb{E}\left[\int_0^{\tau_d(\theta)} e^{-rt} p dt\right]$ . It is the expected discounted value of the CDS premium paid to the protection seller. Hence,  $P(\delta)$  is a contingent claim that pays p before bankruptcy and zero at liquidation. Given the equity's decisions, the value of the premium leg is:

$$P(\delta) = \begin{cases} \frac{p}{r} \left( 1 - P_d^i(\delta) \right), & \text{if } \delta \ge \delta_i; \\ \frac{p}{r} \left( 1 - P_d^0(\delta) \right), & \text{if } \delta_d < \delta < \delta_i \end{cases}$$

In sum, a CDS contract has an expected value of  $C(\delta) - P(\delta)$  to the protection buyer and  $P(\delta) - C(\delta)$  to the protection seller.

Let  $T(\theta, c_B)$  be the collection of thresholds  $\{\delta_i, \delta_n, \delta_d\}$  as functions of the CDS position  $\theta$  and the coupon  $c_B$ , and  $\theta(c_B)$  be the debt's optimal CDS position in response to  $c_B$ , then the pricing condition (2) can be written as

$$C(\delta_0; T(\theta(c_B), c_B)) = P(\delta_0; T(\theta(c_B), c_B)).$$
(3)

Compared to (2), condition (3) explicitly states that the endogenous CDS premium capitalizes the market expectation regarding the impact of the optimal CDS position  $\theta(c_B)$  on the equity's ex-post investment and bankruptcy decisions. Moreover, since the creditors choose the position after the debt issuance, the pricing equation (3) treats the coupon  $c_B$  as a parameter.

### 3.2 Valuation of Corporate Securities

#### 3.2.1 Debt Value and the Optimal CDS Position

Given a CDS position  $\theta$ , the debt has a reservation value of  $R(\delta) = \min\left(\frac{c_B}{r}, \theta + L\delta\right)$ . The reservation value is the sum of the liquidation value and the CDS payment, and it is capped at the contractual principal  $c_B/r$  even when the liquidation value is sufficiently high. Thus,  $R(\delta)$  is the value of the debt holders in a negotiation breakdown, and it constitutes the minimal amount that the debt holders must receive to accept the equity's take-it-or-leave-it offer. It also specifies that the availability of the CDS contracts improves the debt holders' outside option and, thus, their bargaining position in renegotiation.

Let  $b(\delta)$  be a contingent claim that represents the present value of future cash flows accruing

to debt holders, including the payment made by the CDS protection seller upon liquidation. The standard argument implies that  $b(\delta)$  must satisfy the ordinary differential equation (ODE):

$$rb(\delta) = s(\delta) + (\mu + i(\delta))\delta b'(\delta) + \frac{1}{2}\sigma^2 \delta^2 b''(\delta).$$
(4)

The left-hand side is the required return on the claim. The right-hand side is the debt service plus the expected capital gain from holding this claim, given the debt's anticipation of the equity's ex-post optimal investment policy. Because the equity holders have full bargaining power, the debt payoff is pushed down to its reservation value during renegotiation:  $b(\delta) = R(\delta) = \theta + L\delta$ . Substituting this into (4), we have the optimal debt service function:

$$s(\delta) = \begin{cases} c_B, & \text{if } \delta > \delta_n; \\ r\theta + (r - (\mu + i))L\delta, & \text{if } \delta_i \le \delta \le \delta_n; \\ r\theta + (r - \mu)L\delta, & \text{if } \delta_d < \delta < \delta_i. \end{cases}$$
(5)

That is, the interest payment equals the flow of the reservation value in a private workout. It follows that the contingent claim has a closed-form solution:

$$b(\delta) = \begin{cases} \frac{c_B}{r} + \left(\theta + L\delta_n - \frac{c_B}{r}\right) \left(\frac{\delta}{\delta_n}\right)^{z_1}, & \text{if } \delta > \delta_n; \\ \theta + L\delta, & \text{if } \delta_d < \delta \le \delta_n. \end{cases}$$
(6)

Here, the first line is the present value of the default-free coupon and the expected change in value from renegotiation, and the second line represents the reservation value.

We define  $B(\delta) \equiv b(\delta) - C(\delta)$  as the value of a debt contract. The idea is that the debt value, by definition, accounts only for the interest payment and the liquidation value paid out from the firm's assets. The claim  $b(\delta)$  captures the interest payment before bankruptcy. However, it includes the CDS coverage from the protection seller at liquidation as well.<sup>15</sup> Therefore, we subtract the expected injection from the third party to obtain the value of the debt.

Debt holders hold a portfolio of a debt contract, with value  $b(\delta) - C(\delta)$ , and a CDS contract,

<sup>&</sup>lt;sup>15</sup>More precisely, for  $\delta > \delta_d$ , the present value of the interest payments  $\theta + L\delta$  is paid out of the firm's cash flows. At  $\delta_d$ ,  $\theta$  is paid out from the CDS contract and  $L\delta_d$  comes from the firm's liquidated assets. The transfer at bankruptcy captures the *cash settlement* procedure in an actual credit event.

with value  $C(\delta) - P(\delta)$ . The portfolio has a *net payoff*  $b(\delta) - P(\delta)$ :

$$b(\delta) - P(\delta) = \begin{cases} \frac{c_B}{r} + \left(\theta + L\delta_n - \frac{c_B}{r}\right) \left(\frac{\delta}{\delta_n}\right)^{z_1} - \frac{p}{r} \left(1 - P_d^i(\delta)\right), & \text{if } \delta > \delta_n; \\ \theta + L\delta - \frac{p}{r} \left(1 - P_d^i(\delta)\right), & \text{if } \delta_i \le \delta \le \delta_n; \\ \theta + L\delta - \frac{p}{r} \left(1 - P_d^0(\delta)\right), & \text{if } \delta_d < \delta < \delta_i. \end{cases}$$

At time 0, the debt holders choose  $\theta$  to maximize their portfolio value

$$\theta(c_B) = \arg \max_{\theta \in [0,\infty)} \left\{ b(\delta_0; \theta, T(\theta, c_B)) - P(\delta_0; \theta, T(\theta, c_B)) \right\},\$$

given the contractual coupon  $c_B$ , the rational anticipation of the equity's ex-post decision  $T(\theta, c_B)$ , and the competitive CDS pricing (3). Note that at time 0, the net payoff of the debt holders equals the value of the debt because of the pricing condition (3). Moreover, CDS is a non-redundant security because of the strengthened bargaining position that it produces to counter strategic default. To see this, the debt value,  $B(\delta) = L\delta + \theta \left(1 - P_d^0(\delta)\right)$  on  $(\delta_d, \delta_n)$ , is strictly increasing in the CDS position. In contrast to the debt value in Leland (1994), which is independent of the CDS trading, the debt's portfolio here consists merely of two non-interacting securities.<sup>16</sup>

#### 3.2.2 Equity Value and the Endogenous Decisions

The equity value  $E(\delta)$  satisfies the Hamilton-Jacobi-Bellman equation

$$rE(\delta) = \max_{i_t \in \{0,i\}, \delta_d, \delta_n} \left\{ (1-\tau)(\delta - s(\delta)) - \phi i_t \delta + (\mu + i_t) \, \delta E'(\delta) + \frac{1}{2} \sigma^2 \delta^2 E''(\delta) \right\}.$$
(7)

The left-hand side is the required return on equity, which equals the sum of expected dividend net of the investment cost and the expected increment in the equity value on the right-hand side. In (7), the debt service is given by (5). The maximization with respect to  $i_t$  yields  $i(\delta) = i$  if  $E'(\delta) \ge \phi$ , and  $i(\delta) = 0$  otherwise. Since  $E'(\delta)$  is the increase in equity value when the firm invests at  $\delta$ , a unique threshold  $\delta_i$  satisfying  $E'(\delta_i) = \phi$  characterizes the ex-post optimal investment. Given  $\{\delta_d, \delta_i, \delta_n\}$ , the equity value takes the following form. For  $\delta_d < \delta < \delta_i$ , the firm restructures its

<sup>&</sup>lt;sup>16</sup>Formally, to obtain Leland (1994), we can set  $\delta_n = \delta_d$  in (6) to remove strategic default. Then,  $B(\delta) = b(\delta) - C(\delta)$  becomes  $B(\delta) = \frac{c_B}{r} + \left(L\delta_d - \frac{c_B}{r}\right) \left(\frac{\delta}{\delta_d}\right)^{z_1}$  and the market debt is independent of the CDS.

debt continuously and does not invest. We have

$$E(\delta) = (1-\tau) \left( U_0 \delta - (\theta + L\delta) \right) + (1-\tau) \left( \theta + L\delta_d - U_0 \delta_d \right) P_d^0(\delta) + (1-\tau) \Pi \delta_i \Gamma_0(\delta, \delta_d), \tag{8}$$

where  $\Gamma_0(\delta, \delta_d)$  is in the Appendix. On the right-hand side of (8), the first term is the present value of the default-free dividend for the non-investing firm and the second term is the value of the bankruptcy option. We depart from Leland (1994) in that upon bankruptcy, the equity saves the credit protection-dependent debt service flow  $\theta + L\delta_d$  instead of the regular coupon  $c_B$ . The last term is the value of the investment option, which captures both the benefit from investment driven by the increase in the asset growth rate and the loss of the investment option when the firm goes bankrupt.

For  $\delta_i \leq \delta \leq \delta_n$ , the firm invests and restructures its debt continuously. The equity value is

$$E(\delta) = (1-\tau) \left( U_i \delta - (\theta + L\delta) \right) + (1-\tau) \left( \theta + L\delta_d - U_0 \delta_d \right) P_d^i(\delta) + (1-\tau) \Pi \delta_i \Gamma_i(\delta_i, \delta_d) \left( \frac{\delta}{\delta_i} \right)^{z_1}, \quad (9)$$

where  $\Gamma_i(\delta_i, \delta_d)$  is in the Appendix. In (9), the first term is the present value of the default-free dividend for the investing firm and the second term captures the value of the bankruptcy option. Note that as the fundamental  $\delta_t$  deteriorates and crosses  $\delta_i$ , the equity holders stop investing. This implies a reduction in the growth rate of cash flows and an increased likelihood of losing the investment opportunities. Thus, the last term represents the value of the option to reduce investment.

Outside the renegotiation region  $\delta > \delta_n$ , the firm invests and pays the contractual coupon  $c_B$ . The equity value is

$$E(\delta) = (1 - \tau) \left( U_i \delta - \frac{c_B}{r} \right) + (1 - \tau) \left( \theta + L \delta_d - U_0 \delta_d \right) P_d^i(\delta) + (1 - \tau) \Pi \delta_i \cdot \Gamma_i(\delta_i, \delta_d) \left( \frac{\delta}{\delta_i} \right)^{z_1} + (1 - \tau) \left( \frac{c_B}{r} - (\theta + L \delta_n) \right) \left( \frac{\delta}{\delta_n} \right)^{z_1}.$$
 (10)

Similarly, the first and second terms are the present value of the default-free dividend and the value of the default option respectively, and the third term is the value of the option to reduce investment. Additionally, the last term represents the value of the renegotiation option stemming from the limited commitment of the equity holders to pay the contractual debt service. The value depends on the optimally chosen strategic default time and the bargaining position of the CDS-protected debt holders.

The ex-post decisions of the equity holders are as follows. The smooth-pasting of (9) and (10) at  $\delta_n$  delivers the endogenous renegotiation threshold  $\delta_n$  in closed-form:

$$\delta_n = \frac{z_1}{z_1 - 1} \left(\frac{c_B}{r} - \theta\right) \frac{1}{L}.$$
(11)

Thus, the introduction of a CDS market reduces the incentive for the equity holders to default strategically since an increase in the creditors' position in the CDS contract improves their bargaining power. In other words, CDS trading commits the equity holders to pay the regular debt service more often. Importantly, our result generalizes the insight of the empty creditor problem in Bolton and Oehmke (2011) to a dynamic contingent claims model with current and future investment opportunities.<sup>17</sup>

The endogenous investment threshold  $\delta_i$  solves the optimality condition  $E'(\delta_i) = \phi$ , and the bankruptcy threshold  $\delta_d$  satisfies the smooth-pasting condition  $E'(\delta_d) = 0$ . We cannot derive the two endogenous thresholds  $\delta_i$  and  $\delta_d$  in closed-form because they simultaneously solve nonlinear equations, but we can establish the monotonicity of the thresholds in the debt's CDS position.

**Proposition 1.** The equity's expost optimal investment threshold  $\delta_i$  and bankruptcy threshold  $\delta_d$  are strictly increasing in the debt's CDS protection  $\theta$  and have a fixed ratio  $\delta_i/\delta_d$ . Taken together, these results imply that the non-investment region  $(\delta_d, \delta_i)$  expands in  $\theta$ .

The proposition states our main qualitative result: the firms with traded CDSs face worsened debt overhang problems through the empty creditor channel. With the opportunity to purchase the credit derivative, the empty creditor increases the likelihood of bankruptcy. In fact, we show in the proof  $\theta > 0$  implies  $\delta_d > 0$ . The increased chance of bankruptcy affects the distribution of overhang: the debt holders can capture the benefit of equity-financed investments more frequently with the presence of the CDS market, and more so when the firm's distance-to-default is short. The overhang becomes more severe than the non-CDS firms when the debt holders can take a stronger bargaining position by off-loading credit risks. Therefore, as  $\theta$  increases,  $E'(\delta)$  decreases

$$\lim_{\delta \downarrow \delta_n} s(\delta) - \lim_{\delta \uparrow \delta_n} s(\delta) = (c_B - r\theta) \left( 1 - \frac{z_1}{z_1 - 1} \frac{r - (\mu + i)}{r} \right) > 0, \text{ and } \lim_{\delta \downarrow \delta_i} s(\delta) - \lim_{\delta \uparrow \delta_i} s(\delta) = -i \cdot L\delta_i < 0.$$

<sup>&</sup>lt;sup>17</sup>Under the renegotiation policy (11), the debt service function (5) must exhibit discontinuities at  $\delta_n$  and  $\delta_i \in (\delta_d, \delta_n)$ . To see this, note that

The discontinuities imply that the dividend paid to equity jumps upward at  $\delta_n$ . This reflects the concessions made by the debt holders when the equity initiates the private workout. When the equity stops investing at  $\delta_i$ , the dividend jumps downward because the equity holders need to compensate the debt holders for the slow-down in the growth rate of the fundamental grows at a lower rate. Hence, the debt service jumps upward.

and  $D'(\delta)$  increases, which in turn imply a larger non-investment region. Consequently, the equity holders reduce investment earlier and the empty creditor problem exacerbates the under-investment problem.

In our model, the firm takes  $i_t = 0$  only in the renegotiation region  $(\delta_d, \delta_i)$ . Intuitively, the equity may reduce investment when the firm's assets deteriorate. However, as investment creates persistent values, the equity holders would prefer to negotiate down the debt service initially and keep the investment alive as the firm becomes financially distressed. Technically, our setup with binary investment levels drives this feature.<sup>18</sup>

The recent literature on the real effects of CDS trading has not discussed the possibility of debt overhang. Our model formally embeds the empty creditor problem formalized by Bolton and Oehmke (2011). In that model, there is a single project to be financed by debt. The strategic benefit of the CDS contract in reducing the limited commitment frictions increases the ex-ante debt value of the firm. The real impact is positive: the availability of CDS contracts allows the firm to finance a broader set of projects. However, the paper does not discuss the connection between the CDS contracts and future investment opportunities. Danis and Gamba (2016) study the empty creditor problem in a dynamic structural model. The firm faces external financial constraints and is financed with a one-period debt that matures before the next investment opportunity arrives. As in Hennessy and Whited (2005), the firm makes financing and investment decisions simultaneously, and the CDS trading does not change the distribution of overhang.<sup>19</sup> In contrast, our model features under-investment driven by CDS trading through the empty creditor channel.

#### 3.3 Benchmarks

#### 3.3.1 Benchmark: Non-CDS Firms

Suppose that there are no CDS contracts that reference the firm's debt. We can thus take  $\theta = 0$ . In this case, the equity holders never go bankrupt  $\delta_d = 0$ . The reason is as follows. From (5), the debt service at  $\delta$  becomes either  $s(\delta) = (r - \mu)L\delta = (1 - \alpha)(1 - \tau)\delta$  for non-investing firms or

<sup>&</sup>lt;sup>18</sup>The formulation allows us to characterize the renegotiation threshold in closed-form in order to study the impact of the CDS market. In an earlier version of the paper, we showed that the renegotiation threshold takes the same form as (11) and debt overhang is increased by CDS trading in a one-shot real option investment framework, for example, Hackbarth and Mauer (2012). Our results should hold in a continuous investment setup, for example, Hennessy (2004). In that case, we expect the firm will under-invest even outside the renegotiation region.

<sup>&</sup>lt;sup>19</sup>Along with Danis and Gamba (2016), we assume the maturity of CDS matches the debt maturity. In practice, CDS contracts typically have a five-year maturity. The average debt maturity in Saretto and Tookes's (2013) sample of S&P 500 firms is 8.68 years. The "maturity mismatch" between CDS contracts and debt suggests that illiquid and long-term debt investors may face "rollover risk" that depends on the liquidity of the CDS markets. These factors may affect the strategic benefits of CDS the debt holders anticipate and the debt overhang effect.

 $s(\delta) = (1 - \alpha)(r - (\mu + i))\frac{1 - \tau}{r - \mu}\delta$  for investing firms. Regardless of the investment decision, the debt service specifies a linear sharing rule of the cash flows between the debt and equity in the absence of credit protection. Therefore, the equity holders absorb no losses and thus never go bankrupt.<sup>20</sup> From (11),  $\delta_n = \frac{z_1}{z_1 - 1} \frac{c_B}{rL}$  is the renegotiation threshold.

#### **Proposition 2.** Under Assumption 1, there is no debt overhang without the CDS market.

The proposition implies that we can measure the overhang by comparing the asset value of non-CDS firms and that of CDS firms. During a private workout, the present value of strategic debt service is L per unit of  $\delta$ . Assumption 1 guarantees that the present value of dividends with investment net of the investment cost is positive. In fact, it ensures that the benefit of investment shared by the debt holders is sufficiently small. Therefore, it is optimal for the equity holders to invest at all times without the CDS market.<sup>21</sup>

#### 3.3.2 Benchmark: Value-Maximizing Investment

Unlike the case of no CDS benchmark, the value-maximizing benchmark opens the CDS market but assumes that equity holders choose an investment policy to maximize the firm value after the debt is in place. The comparison of the first-best firm value to the firm value with equity-maximizing investment policy (second-best firm, Section 3.2.2) allows us to derive the *agency cost of CDS* that captures the investment inefficiency associated with CDS trading. Proposition 2 implies the agency cost contains only the inefficiency induced by the availability of CDSs.

Suppose the equity holders commit to a first-best investment policy that maximizes the firm value  $V(\delta) = E(\delta) + B(\delta)$  after the debt is in place. The firm value must satisfy the ODE

$$rV(\delta) = \max_{i_t \in \{0,i\}} \left\{ (1-\tau)\delta - \phi i_t \delta + \tau s(\delta) + (\mu + i_t)\delta V'(\delta) + \frac{1}{2}\sigma^2 \delta^2 V''(\delta) \right\}.$$

Similar to (7), the required return on the firm is the sum of the after-tax cash flows net of investment costs, the tax saving, and the expected capital gain. The key difference with (7) is that the maximization problem involves the investment level only, and the equity holders still choose the renegotiation and bankruptcy threshold to maximize the market value of equity ex-post.

<sup>&</sup>lt;sup>20</sup>When  $\theta = 0$ , then the left-hand side of (18), the equation that pins down the bankruptcy threshold, scales with  $\delta_d$  and hence  $\delta_d = 0$  is the unique solution.

<sup>&</sup>lt;sup>21</sup>For the non-CDS firms, under-investment may occur without bankruptcy because investment increases the growth rate of the cash flows and hence the chance that the debt holders is paid the higher regular coupon  $c_B$ . Technically, for any investment threshold,  $E'(\delta_i)$  is a constant. Assumption 1 guarantees that  $E'(\delta_i)$  is sufficiently large when  $\delta_i \to 0$ , and this is shown analytical in the appendix. See Sundaresan and Wang (2007) and Pawlina (2010) for related discussions on under-investment with renegotiable debt.

The maximization with respect to  $i_t$  yields  $i^{FB}(\delta) = i$  if  $V'(\delta) \ge \phi$  and  $i^{FB}(\delta) = 0$  if  $V'(\delta) < \phi$ . This condition, together with the smooth-pasting at bankruptcy  $E'(\delta_d) = 0$  and the smooth-pasting of (9) and (10) at renegotiation, determine the policies  $\{\delta_d, \delta_i, \delta_n\}$  when the equity holders commit to the first-best investment rule. As the first-best investment decision internalizes the debt value, there is less under-investment under the equity's commitment.<sup>22</sup> In fact, we have the following result.

## **Proposition 3.** Under Assumption 1, first-best firms invest all the time until bankruptcy: $\delta_i = \delta_d$ .

With the equity's commitment to the value-maximizing investment until bankruptcy, we can derive the associated equity value from (8), (9), and (10) by sending  $\delta_i \to \delta_d$ . Outside the renegotiation region  $\delta > \delta_n$ , the equity value is

$$E(\delta) = (1 - \tau) \left[ \left( U_i \delta - \frac{c_B}{r} \right) + \left( \frac{c_B}{r} - (\theta + L\delta_n) \right) \left( \frac{\delta}{\delta_n} \right)^{z_1} + (\theta + L\delta_d - U_i\delta_d) \left( \frac{\delta}{\delta_d} \right)^{z_1} \right]; \quad (12)$$

and in the renegotiation region and before bankruptcy  $\delta_d < \delta \leq \delta_n$ ,

$$E(\delta) = (1 - \tau) \left[ (U_i \delta - (\theta + L\delta)) + (\theta + L\delta_d - U_i \delta_d) \left(\frac{\delta}{\delta_d}\right)^{z_1} \right].$$
(13)

The usual smooth-pasting conditions of the equity values (12) and (13) at  $\delta_n$  and  $E'(\delta_d) = 0$ characterize the endogenous renegotiation and bankruptcy threshold,  $\delta_n = \frac{z_1}{z_1-1} \left(\frac{c_B}{r} - \theta\right) \frac{1}{L}$  and  $\delta_d = \frac{z_1}{z_1-1} \frac{1}{U_i-L} \theta$ . Note that bankruptcy occurs on the equilibrium path because the bankruptcy decision maximizes the equity value. Moreover, the bankruptcy cost affects the debt service in outof-court restructurings and hence the bankruptcy decision. This is in contrast to Leland (1994), who does not allow debt renegotiation; thus, the bankruptcy threshold is affected by contractual coupon but not the bankruptcy cost.

*Remark.* For notations, we denote  $V_{CDS}$  as an equity-maximizing (second-best) CDS firm value,  $V_{FB}$  as a value-maximizing (first-best) CDS firm value, and  $V_0$  as the value of a non-CDS firm. We may drop the subscripts whenever the context poses no ambiguity.

<sup>&</sup>lt;sup>22</sup>To see this, fix a  $\delta_d$ . The optimality condition for an interior solution can be written as  $V'(\delta_i^{FB}) = E'(\delta_i^{FB}) + b'(\delta_i^{FB}) - C'(\delta_i^{FB}) = \phi$ . Note that  $b'(\delta) - C'(\delta) > 0$  for all  $\delta > \delta_d$ : On the one hand, the debt faces less credit risks when the firm has a stronger fundamental and so a higher debt payoff. On the other hand, the expected payout of the credit derivative decreases as the distance-to-default increases. It follows that  $E'(\delta_i) = \phi > E'(\delta_i^{FB})$  and since the equity value is convex, we have  $\delta_i > \delta_i^{FB}$ .

#### **3.4 Optimal Capital Structure**

The optimal financial leverage balances the benefit of debt tax shield and the cost of bankruptcy. At time t = 0, the equity holders choose a coupon to maximize the ex-ante equity value (firm value)

$$\max_{c_B} V(\delta_0; \theta(c_B), T(\theta(c_B), c_B)),$$

given their ex-post optimal investment and financial policies, and the debt's optimal hedging strategy. Denote  $c_B^*$  as the optimal coupon and  $\theta^* = \theta(c_B^*)$  as the optimal CDS position given the optimal coupon. For any coupon, CDS position, and the associated endogenous thresholds, we define the firm's market leverage ratio and credit spread, respectively, as

$$ML(\delta_0) = \frac{B(\delta_0; \theta, T(\theta, c_B))}{E(\delta_0; \theta, T(\theta, c_B)) + B(\delta_0; \theta, T(\theta, c_B))}, \text{ and } CS(\delta_0) = \frac{c_B}{B(\delta_0; \theta, T(\theta, c_B))} - r.$$

The ratio of the CDS premium to the notional amount  $p/\theta(c_B)$  defines the CDS spread. These objects are defined analogously for the first-best CDS firm in which the equity holders optimize the capital structure given their commitment to the first-best investment policy once the debt is in place. Following the discussion in the first-best benchmark, the CDS-induced agency cost of debt as  $AC(\delta_0) = V_{FB}(\delta_0)/V_{CDS}(\delta_0) - 1$ . Lastly, as in Leland (1994), our model features scale invariance.

**Proposition 4** (Scale Invariance). Under the optimal choice, the thresholds  $T = \{\delta_d, \delta_i, \delta_n\}$ , the CDS position  $\theta^*$ , the coupon  $c_B^*$ , and the market values  $E(\delta_0)$  and  $B(\delta_0)$  are functions of homogeneous of degree one in the initial cash flows  $\delta_0$ . It follows that  $ML(\delta_0)$ ,  $CS(\delta_0)$ , and  $AC(\delta_0)$ are homogeneous of degree zero in  $\delta_0$ .

Although CDS trading increases the debt's payoff linearly in the renegotiation states, the additional interest  $r\theta$  scales with the bankruptcy threshold. The implication is that these thresholds are still independent of the firm scale. Therefore, the optimal leverage ratio  $ML(\delta_0)$ , credit spread  $CS(\delta_0)$ , and agency cost of CDS  $AC(\delta_0)$  are independent of the initial cash flows  $\delta_0$ .

## 4 Quantitative Analysis

In this section, we calibrate the model to parameters that match previous studies. We set the risk-free interest rate to r = 5%, the baseline risk-neutral drift to  $\mu = 1\%$ , the volatility of the



Figure 1: The endogenous investment and default policy and the debt value. Left Panel: The solid line is  $\delta_i$  and the dashed line is  $\delta_d$ . The dotted line shows the bankruptcy threshold  $\delta_d$  for the first-best CDS firms. Right Panel: The solid lines are the market values of debt at time 0 and the dashed lines are the expected payments to the CDS seller, with thick lines indicating the second-best firms and thin lines indicating the first-best firms. The parameters are r = 5%,  $\mu = 1\%$ ,  $\sigma = 25\%$ , i = 2.5%,  $\tau = 20\%$ ,  $\alpha = 35\%$ . We choose  $\phi = 12.5$ , which gives  $\Pi/U_0 = 62.5\%$ , and set the coupon at the optimum:  $c_B^* = 12.7$ . Under the optimal CDS  $\theta^* = 131.06$ , the renegotiation threshold is  $\delta_n = 5.39$ .

cash-flow shock to  $\sigma = 25\%$ . For the investment option, we set i = 2.5%, so the growth rate of unlevered firms is  $\mu + i = 3.5\%$ , which is comparable to the simulated growth rate of 3.31% in He's (2011) analysis of dynamic agency and debt overhang. The investment cost parameter is chosen to be  $\phi = 12.5$ , which implies that the firm reinvests  $\phi i = 31.25\%$  of cash flows in the investment region, and an investment value of  $\Pi/U_0 = 62.5\%$ . We calibrate the effective tax rate (including personal taxes) to  $\tau = 20\%$  and the bankruptcy cost to  $\alpha = 35\%$ . This choice satisfies Assumption 1.

### 4.1 The Impact of CDS Trading and Calibration

Figure 1 graphs the impact of CDS trading on decisions and the debt's net payoff under a fixed capital structure. The left panel, which depicts the endogenous investment and bankruptcy policy, verifies Proposition 1 and 2. In particular, it shows that the equity holders of a CDS firm choose higher investment (blue solid) and bankruptcy (red dashed) thresholds as  $\theta$  increases, resulting in an expansion of the under-investment region  $(\delta_d, \delta_i)$ . For non-CDS firms, this is a special case with  $\theta = 0$ . We have  $\delta_i(\theta = 0) = \delta_d(\theta = 0) = 0$  because of the linear sharing of cash flows in renegotiation and the absence of bankruptcy on the equilibrium path. Holding fixed the capital structure, the



Figure 2: The optimal capital structure and the debt's optimal CDS position. For CDS firms, the optimal leverage ratio is  $ML(\delta_0) = 58.15\%$  and the credit spread is  $CS(\delta_0) = 117$  bps; for non-CDS firms, the optimal leverage ratio is 37.04% and the credit spread is 384 bps. The thin lines in both panels are for first-best CDS firms. The percentage difference in values between the first-best and second-best CDS firms under the respective optimal leverage,  $AC(\delta_0) = 0.376\%$ , captures the debt overhang cost. The parameters are r = 5%,  $\mu = 1\%$ ,  $\sigma = 25\%$ , i = 2.5%,  $\phi = 12.5$ ,  $\tau = 20\%$ ,  $\alpha = 35\%$ , and initial cash flows  $\delta_0 = 10$ .

bankruptcy threshold associated with the equity's commitment to the first-best investment policy (thin dotted) mostly overlaps with the second-best bankruptcy threshold. This value-maximizing benchmark reveals that the key driver for bankruptcy is the CDS trading rather than the inability for the equity holders to switch to an optimal investment level.

The right panel of Figure 1 depicts the time-0 market value of debt (the solid lines) and the CDS price  $C(\delta_0)$  (the dashed lines). The CDS premium is increasingly costly in the hedging position because a higher  $\theta$  accelerates the bankruptcy time. In anticipation of an earlier bankruptcy, the protection sellers charge a higher CDS premium. The debt's net payoff is concave in  $\theta$  because an excessive CDS position induces a price that outweighs the strategic benefit. Under the first-best investment decisions that internalize the debt, the asset-growth rate becomes higher and the firm liquidation becomes less likely, thus implying a higher debt value (thin black solid line) and less costly CDS protection (thin red dashed line).

Figure 2 illustrates the firm's optimal capital structure and the debt's optimal CDS position. The left panel of Figure 2 shows that  $\theta(c_B)$  is increasing in the contractual coupon  $c_B$  for both equity- and value-maximizing firms.<sup>23</sup> Facing a highly levered firm, it is beneficial for the creditors to strengthen their bargaining position because of a stronger strategic default incentive.

 $<sup>^{23}</sup>$ The formal argument for this linear equilibrium is provided in the proof of Proposition 4.

The right panel of Figure 2 shows the ex-ante optimal coupon that maximizes firm value. Non-CDS firms (black dotted-dashed line) issues an optimal coupon of  $c_B^* = 11.5$  that exhausts the debt capacity:  $c_B^* = c_B^{max} \equiv \arg \max_{c_B} B(\delta_0; \theta = 0, T(\theta = 0, c_B))$ . The intuition is that for any coupon larger than  $c_B^*$ , the equity initiates a private workout immediately once the firm starts its operation. It follows that the creditors are unwilling to lend more than the debt capacity.<sup>24</sup>

CDS firms (blue solid) sets an optimal coupon of  $c_B^* = 12.7$  and remove the value-maximizing CDS firms (thin red) promises  $c_B^* = 13.3$ . Compared to a non-CDS firm, a CDS firm has an expanded debt capacity because of the strategic benefit enjoyed by the credit-protected debt holders.<sup>25</sup> The debt capacity is more substantial when the equity holders could commit to a value-maximizing investment policy that internalizes the debt value: a higher asset growth rate implies a higher debt value.

Table 1 reports the calibration of the model. In panel A, we report the model-implied financial variables and the decision thresholds under the optimal capital structure. Because of the CDS-induced expansion in debt capacity, the optimal leverage ratio increases from 37.04% to 58.15% for the equity-maximizing firm and to 60.31% for the value-maximizing firm. Additionally, the CDS firm faces non-trivial under-investment:  $\delta_i = 3.83 > \delta_d = 2.77$ ; and the bankruptcy time for a value-maximizing firm is slightly earlier than an equity-maximizing firm:  $\delta_d^{FB} = 2.95 > \delta_d = 2.77$ . As seen in the left panel of Figure 1, for a fixed coupon, the bankruptcy threshold under first-best investment is smaller than that under second-best investment. However, the first-best firm has a higher leverage, which accelerates the equity's ex-post optimal bankruptcy time.

Additionally, the model-implied credit spread is the same as the CDS spread  $p/\theta^* = 116.88$  basis points. The reason is that the primary driving factor for a non-zero CDS-bond basis is the liquidity of the bond and CDS market, not the investment opportunities of the firm.<sup>26</sup> Moreover, the optimal hedge ratio is  $\frac{\theta^*}{c_B^*/r} = 51.60\%$ .<sup>27</sup> Finally, the increase in the debt value implies a lower credit spread for CDS firms. With the introduction of the CDS market equity holders must

<sup>&</sup>lt;sup>24</sup>Formally, the firm promises a coupon  $c_B^*$  such that  $\delta_n(c_B^*) = \frac{z_1}{z_1-1} \frac{c_B^*}{rL} = \delta_0$  in the absence of the CDS market. The debt value as a function of coupon has the same shape as the bank debt discussed in Section 3.1 of Hackbarth, Hennessy, and Leland (2007).

<sup>&</sup>lt;sup>25</sup>As in Leland (1994),  $c_B^* < c_B^{max} \equiv \arg \max_c B(\delta_0; \theta(c_B), T(\theta(c_B), c_B))$  since a coupon rate higher than the optimal one induces a higher bankruptcy cost.

<sup>&</sup>lt;sup>26</sup>See, for example, Longstaff, Mithal, and Neis (2005) for a reduced-form model that captures CDS spread as a measure of bankruptcy component of bond spread in illiquid bond markets; and Oehmke and Zawadowski (2015) show that in market equilibrium, negative CDS-bond basis is driven by bond trading costs and disagreement among market participants about the bond's default probability.

<sup>&</sup>lt;sup>27</sup>Using data published by the Depository Trust and Clearing Corporation (DTCC), Danis and Gamba (2018) document an average net notional/debt of 28%. Colonnello et al. (2017) document a comparable average of 32.5%. Oehmke and Zawadowski (2016) report a mean Net CDS/bonds (face value; direct issuance) of 50.3%.

Table 1: Baseline calibration. The parameters are r = 5%,  $\mu = 1\%$ ,  $\sigma = 25\%$ , i = 2.5%,  $\phi = 12.5$ ,  $\tau = 20\%$ ,  $\alpha = 35\%$ , and the initial cash flows  $\delta_0 = 10$ . The investment option increases the value of assets-in-place by  $\Pi/U_0 = 62.5\%$ . The first two columns report the quantitative results for CDS firms. The second-best (SB) investment policy maximizes the equity value and the first-best (FB) investment policy maximizes the firm value. The last column reports the result for non-CDS firms. Panel A reports the optimal initial coupon  $c_B^*$ , the debt's optimal CDS position  $\theta^*$ , the hedge ratio  $\frac{\theta^*}{c_B^*/r}$ , the market leverage  $ML(\delta_0) = \frac{D(\delta_0)}{D(\delta_0) + E(\delta_0)}$ , the quasi-market leverage  $\frac{c_B^*/r}{c_B^*/r + E(\delta_0)}$ , the credit spread  $CS(\delta_0) = c_B^*/D(\delta_0) - r$ , the bankruptcy, investment, and renegotiation thresholds  $(\delta_d, \delta_i, \delta_n)$ , the firm value  $V(\delta_0)$ , and the unlevered asset value  $A(\delta_0)$ . Appendix A provides the expression for  $A(\delta_0)$ . Panel B provides the decomposition of firm value and we report the percentage of firm value contributed by various sources. There, net tax shield (NTS) is net tax shield  $TS(\delta_0)$  minus the bankruptcy cost  $BC(\delta_0)$ . The agency cost of CDS is  $AC(\delta_0) = (V_{FB} - V_{CDS})/V_{CDS} \times 100\%$ . Panel C reports the difference in the quantities relative to non-CDS firms over the value of non-CDS firms (denoted by  $A_0$  or  $V_0$ ), and they satisfy the relation  $\frac{\Delta V(\delta_0}{V_0(\delta_0)} = \frac{\Delta Asset}{V_0(\delta_0)} + \frac{\Delta NTS}{V_0(\delta_0)}$ .

	CDS	Non-CDS firms									
	Equity max. (SB)	Value max. (FB)	Equity max.								
Panel A: Financial variables and endogenous decisions											
Coupon $c_B^*$	12.7	13.3	11.5								
CDS position $\theta^*$	131.06	140.79	-								
Hedge ratio (%)	51.60	52.93	-								
Market leverage $(\%)$	58.15	60.31	37.04								
Quasi-market leverage $(\%)$	63.15	65.35	58.74								
Credit spread (bps)	116.88	120.44	384.62								
$\delta_d/$ $\delta_i/$ $\delta_n$	2.77/3.83/5.39	2.91/2.91/5.49	-/-/10.00								
Firm value $V(\delta_0)$	354.07	355.41	351								
Asset value $A(\delta_0)$	317.93	317.95	325								
Panel B: Value decomposition											
Percentage of firm value from:											
Equity value	41.85%	39.69%	62.96%								
Debt value	58.15%	60.31%	37.04%								
Unlevered asset	89.79%	89.46%	92.59%								
Tax shield	11.24%	11.65%	7.41%								
Bankruptcy cost	1.04%	1.11%	0%								
Net tax shield (NTS)	10.21%	10.54%	7.41%								
Agency Cost of CDS $(\%)$	0.3760										
Panel C: Difference with non-CDS Firms											
Firm value $\Delta V(\delta_0)/V_0(\delta_0)$	0.875%	1.256%									
Unlevered asset $\Delta Asset/V_0(\delta_0)$	2.015%	2.008%									
Net tax shield $\Delta \text{NTS}/V_0(\delta_0)$	2.890%	3.264%									

postpone strategic default, and the spread decreases from 384.62 basis points to 116.88 basis points for the equity-maximizing firm and to 120.44 basis points for the value-maximizing firm.

Despite the adverse effect of under-investment and accelerated bankruptcy, the firm value still increases from  $V_0(\delta_0) = 351$  to  $V_{CDS}(\delta_0) = 354.07$ , which amounts to a 0.875% increase, in the baseline calibration. To better understand the sources of value, Panel B of Table 1 provides a value decomposition. There are a few observations. First, CDS trading induces a significant wealth transfer from the equity holders to the debt holders. Second, the increase in the debt tax shield is the primary driver of the increase in the firm value. In particular, the net tax shield accounts for only 7.41% of the value of a non-CDS firm but 10.21% of the value for a CDS firm. The last three rows of the table ("difference with non-CDS firms") decompose the percentage increase in firm value into the reduction in asset value and the gain in the net tax shield. The percentage change in firm value attributable to the increase in the net tax shield is 2.89%.

Importantly, we quantify the CDS-induced debt overhang cost with two measures. First, we compute the agency cost of CDS that captures the percentage loss in firm value of a CDS firm relative to a value-maximizing CDS firm. In the baseline calibration, the agency cost of CDS is  $AC(\delta_0) = 0.376\%$ . While the magnitude may seem to be small, we emphasize that our agency cost should be interpreted as the debt overhang cost induced by CDS trading. Therefore, our model captures a new dimension of the agency cost of debt.<sup>28</sup> The existing literature that considers long-term debt estimates the debt overhang cost to be 2% in Mauer and Ott (2000), 4.7% in Moyen (2007), and approximately 0.2% to 1% in Hackbarth and Mauer (2012). More recently, Hackbarth, Rivera, and Wong (2017) quantify a 1% of agency cost in a dynamic agency model with short-termism. Taking a simple "adding-up" view, the inception of CDS trading could lead to an 8% to over 50% increase in the debt overhang cost estimates in the literature. Hence, the debt overhang problem implied by the empty creditor channel is non-trivial.

Second, we compute the reduction in the asset value. Inspection of Table 1 gives  $A_0(\delta_0) - A_{CDS}(\delta_0) = 7.07$ . In panel B, we report the difference in asset value between a CDS and a non-CDS firm scaled by the non-CDS firm value. The quantity is  $(A_0(\delta_0) - A_{CDS}(\delta_0))/V(\delta_0) \times 100\% = 2.015\%$ , and it measures the percentage of the non-CDS firm value destroyed due to underinvestment. For completeness, we calculate the percentage change in asset value  $(A_0(\delta_0) - A_{CDS}(\delta_0))/A_0(\delta_0) \times 100\% = 2.18\%$ .<sup>29</sup> Consequently, the reduction in the asset value is relatively

 $<sup>^{28}</sup>$ Section 4.2 reports substantial variations in the agency cost of CDS with respect to investment opportunities and other parameters.

<sup>&</sup>lt;sup>29</sup>The comparison of  $A_{FB}(\delta_0) = 317.95$  and  $A_{CDS}(\delta_0) = 317.93$  seems to suggest the commitment of the first-best

large: the negative impact erodes the gain in the tax shield.

Our observations highlight a two-fold effect of CDS protection on firm value, that is, a trade-off between increased tax benefits and debt overhang. Therefore, the inception of CDS trading is only beneficial from the firm's perspective when the tax shield is sufficiently strong, and it always makes shareholders worse off ex-post.

In sum, our qualitative results are consistent with the empirical evidence provided by the existing literature. Saretto and Tookes (2013) estimate that the increase in debt ratios due to CDS tradings is between 0.9% and 5.5%, with the average market leverage being 13% for non-CDS firms and 18% for CDS firms in their sample. Subrahmanyam et al. (2014) observe increases in leverages with the inception of CDS trading and report that firms with CDS actively traded have the probability of bankruptcies increased from 0.14% to 0.47%. Ashcraft and Santos (2009) find that the CDS market lowers credit spreads for firms with high credit quality, and Kim (2016) documents that firms with strong strategic default incentives have their credit spreads decreased with the introduction of CDS referencing the firm.<sup>30</sup>

### 4.2 Comparative Statics

It is evident that both the profitability of investment opportunities and cash-flow risk affect debt overhang. Perhaps equally important, the greater the liquidation value of the asset, the less incentive equity holders have to invest because debt holders' post-renegotiation cash flows increase with liquidation value. This section provides comparative statics results based on such characteristics. Specifically, we examine the heterogeneous impact of CDS trading for firms with different business risks, bankruptcy costs that reduce liquidation values, and profitability of investment opportunities.

We document several key findings. First, CDS trading results in higher market leverages, smaller equity values, and larger debt values, and the effects are stronger for firms with more efficient investment technologies. Second, CDS-induced debt overhangs are stronger for high-risk firms and for firms with low growth potentials and low bankruptcy costs. Third, firms subject to

investment policy does not improve the asset value. This is misleading because a first-best firm has a higher debt capacity and issues a higher coupon than the corresponding second-best firm. This implies the equity holders will declare bankruptcy earlier ( $\delta_d^{FB} = 2.91 > \delta_d^{CDS} = 2.77$ ), which hurts the asset.

<sup>&</sup>lt;sup>30</sup>As is well-known, Leland (1994) produces the low-leverage puzzle. Under reasonable parameter values, the modelimplied optimal leverage is 60-80%. Under our baseline parameters, Leland's model generates a 57.14% leverage. As we adopt Leland's (1994) framework, we do not aim to match the observed leverage ratios and instead focus on the impact of the inception of CDS trading. Fortunately, the scale-invariance property of our model allows the usual tricks that address the low-leverage puzzle, for example, upward debt restructuring (Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), Strebulaev (2007)), to be embeddable in our framework. We leave the dynamic capital structure decisions for CDS firms for further research.



Figure 3: The heterogeneous impact of CDS trading on high- and low-risk firms. The fixed parameters are r = 5%,  $\mu = 1\%$ ,  $\tau = 20\%$ ,  $\alpha = 35\%$  and the initial cash flow is  $\delta_0 = 10$ . The investment value is measured as  $\Pi/U_0 = (U_i - U_0)/U_0$ . We measure the agency cost of CDS as  $(V_{FB}(\delta_0)/V_{CDS}(\delta_0) - 1) \times 100\%$  and the hedge ratio as  $\theta^*/(c_B^*/r)$ . A high (low) risk firm has an instantaneous volatility of  $\sigma_H = 28\%$  ( $\sigma_L = 21\%$ ) and is plotted in dashed (solid) lines.

strong debt overhang have a reduction of firm value up to 2.5% post-CDS introduction. In the figures below, we vary the investment cost  $\phi \in [2, 16]$  to obtain  $\Pi/U_0$ .

Figure 3 shows the effect of CDS trading on the firm, equity, and debt value (the upper panel), the agency cost of CDS, the hedge ratio, and the credit spread (the lower panel) by varying the investment opportunities. The solid lines are for the high-risk firm,  $\sigma_H = 28\%$ , and the dashed lines are for the low-risk firm,  $\sigma_L = 21\%$ . The upper panel reports the effect of introducing the CDS market as the percentage change in firm value.

There are several implications. First, the debt holders of a high-risk firm hedge less. The intuition comes from an equilibrium price effect: an increase in the cash-flow volatility increases the bankruptcy likelihood; thus, the protection sellers would charge a higher CDS premium. When a firm has poor investment opportunities and is thus more likely to liquidate, the price effect also implies that the debt holders choose a lower hedge to economize the cost of credit protection.

Second, firms that have more profitable investment opportunities benefit more from the CDS trading because they are less subject to debt overhang and default risk. The less costly strategic positions for growth firms allow their debt holders to transfer more wealth from the shareholders. As a result, the CDS-induced increase in debt capacity and tax shield are greater for high-growth firms than for low-growth ones, and the shareholders become worse off.

Importantly, the inception of CDS trading does not always increase the firm value. In particular, the value of a high-risk CDS firm with poor investment opportunities can be 1% to 2% lower than the corresponding non-CDS firm. On the one hand, the volatility-implied price effect forces the debt holders to purchase less credit protection, so the positive impact on the debt capacity is smaller for the high-risk firms. On the other hand, the higher default likelihood implies a stronger debt overhang effect once the firm has CDSs traded. We can observe this from the bottom left panel. The agency cost of CDS has a negative relationship with the profitability of investment opportunities, and the pattern is more pronounced for high-risk firms. Specifically, the agency cost of the high-risk firm can be 0.6-0.7% higher (at  $\Pi/U_0 = 33.3\%$ ) than that of the low-risk firm.

Figure 4 examines the impact of CDS trading on firms with a different degree of bankruptcy cost. The cost parameter is a proxy for the asset intangibility: firms with more tangible assets tend to have a lower bankruptcy cost. The panel for the hedging ratio shows the behavior of the creditors. Note that the threat posed by the shareholders in a private workout is more credible in firms with a high bankruptcy cost. Thus, without the CDS market, the debt's reservation value decreases with the bankruptcy cost. As a result, for firms with high bankruptcy cost, the creditors choose a higher hedge ratio because the CDS contracts provide a more significant strategic benefit.

The heterogeneous strategic benefit of the CDS contract implies that firms with a low liquidation value benefit more from the inception of CDS trading. As the debt capacity and the tax shield increase further, the wealth transfer from the equity to the debt also becomes stronger. In addition, low-growth CDS firms with a high liquidation value have a firm value that is 1% to 2.5% lower than the corresponding non-CDS firms. While the creditors of firms with a low bankruptcy cost hedge less, a high liquidation value still implies that they make less concession in renegotiation and thus the equity chooses a higher bankruptcy threshold. Therefore, the debt overhang effect implies that the loss in value from the inability of the equity to commit to the first-best policy can exceed 1% for firms with less efficient investment technologies. For  $\Pi/U_0 = 33.3\%$ , the agency cost of CDS for firms with a low bankruptcy cost (3%) is twice the cost for the high bankruptcy cost firm (1.5%).



Figure 4: The heterogeneous impact of CDS trading on firms with high-and lowbankruptcy costs. The fixed parameters are r = 5%,  $\mu = 1\%$ ,  $\tau = 20\%$ ,  $\alpha = 35\%$  and initial cash flow  $\delta_0 = 10$ . The investment value is measured as  $\Pi/U_0 = (U_i - U_0)/U_0$ . We measure the agency cost of CDS as  $(V_{FB}(\delta_0)/V_{CDS}(\delta_0) - 1) \times 100\%$  and the hedge ratio as  $\theta^*/(c_B^*/r)$ . A high (low) bankruptcy cost firm has  $\alpha_H = 40\%$  ( $\alpha_L = 30\%$ ) and is plotted in dashed (solid) lines.

### 5 Extensions and Discussions

#### 5.1 Debt Holders' Offers

Consider a situation where debt holders have all the bargaining power in renegotiation. In this case, debt holders can make a take-it-or-leave-it offer to the equity holders in a private workout. The key result in this section is that CDS becomes redundant securities and thus does not affect the firm valuation and real decisions. Intuitively, as the debt holders have all the bargaining power, their outside option, which is affected by their positions in CDS, becomes irrelevant for the bargaining outcomes.

As a first step of the analysis, we assume for the moment that the firm always invests. As the debt has the full bargaining power, the equity will be pushed down to its outside option value, which is zero under limited liability and absolute priority. This implies that for any  $\delta \in [\delta_d, \delta_n]$ , the

equity value is zero and the debt extracts all the cash flows, leading to the debt service function:

$$s(\delta) = \begin{cases} (1 - \tilde{\phi}i)\delta & \text{for } \delta \in [\delta_d, \delta_n]; \\ c_B & \text{for } \delta > \delta_n. \end{cases}$$
(14)

As the debt holders are paid all cash flows of the firm during renegotiation, we expect them to behave as residual claimants. As a consequence, it is the debt holders that effectively choose when to liquidate the firm. To understand the debt's incentives to initiate the formal bankruptcy procedure, first, note that the debt would never declare the firm's bankruptcy when all they receive is the positive cash flows  $(1 - \tilde{\phi}i)\delta$  in the private workout. This implies that the only incentive for the debt to liquidate the firm must come from the collection of CDS coverage  $\theta$ .

However, since the CDS contract does not affect the debt's bargaining position and given that the contract is fairly priced, the ex-ante debt value only depends on the CDS position through its effect on liquidation time. The presence of the bankruptcy cost then forces the debt holders to reduce their CDS trading to minimize the possibility of liquidation. This logic yields the following proposition.

**Proposition 5.** When debt holders can make a take-it-or-leave-it offer to equity holders, the optimal debt service function is given by (14). Debt holders do not hedge ( $\theta = 0$ ), and never liquidate the firm ( $\delta_d = 0$ ). Consequently, the inception of CDS trading does not worsen debt overhang when equity holders have no bargaining power.

When the equity holders have the flexibility to choose the investment level, the benefits of investment do not accrue to the debt because there is no bankruptcy on the equilibrium path. In fact, as the debt holders behave like the residual claimants, they have even lower incentives to hedge against the firm's credit risk with CDS contracts as they anticipate that the equity will adopt an inefficient investment policy.

#### 5.2 Renegotiation Frictions

This subsection analyzes the impact of renegotiation frictions on the firm's financial and investment policies with the presence of the CDS market. As in the base case, equity holders make take-itor-leave-it offers. We capture renegotiation frictions as an exogenous probability  $q \in [0, 1)$  of negotiation failure. When renegotiation fails, the firm liquidates, in which case debt holders claim  $\theta + L\delta$  and equity holders get nothing. In other words, the equity can capture the value  $E(\delta)$  with probability 1 - q in a private workout, and q reflects the magnitude of renegotiation costs, which are borne by equity holders. The modeling advantage of a proportional renegotiation cost is that as the equity value scales with q in the renegotiation region, the magnitude of q has no direct consequence on the bankruptcy decision. It allows us to isolate the direct effect of q on CDS price and focus on the strategic benefit of hedging.<sup>31</sup>

For the valuation, renegotiation frictions have no direct impact on the debt portfolio, and the market value of debt is still given by (6). The equity value satisfies

$$rE(\delta) = \max_{i_t \in \{0,i\}, \delta_d, \delta_n} \left\{ (1-\tau)(\delta - s(\delta)) - \phi i_t \delta + (\mu + i_t) \, \delta E'(\delta) + \frac{1}{2} \sigma^2 \delta^2 E''(\delta) \right\}.$$
(15)

Denote  $E^q(\delta)$  as the solution to equation (15). Proportionality implies that for  $\delta \in [\delta_d, \delta_n]$ ,  $E^q(\delta) = (1-q) \cdot E(\delta)$  with  $E(\delta)$  given by (9) or (8). It follows that the bankruptcy threshold  $\delta_d$  is independent of q because of the smooth-pasting condition  $E^{q'}(\delta_d) = (1-q) \cdot E'(\delta_d) = 0$ , which holds given  $E'(\delta_d) = 0$ . Moreover, the investment threshold  $\delta_i$  is also independent of q because  $E^{q'}(\delta) = (1-q)\phi$  if and only if  $E'(\delta_i) = \phi$ , for  $\delta_i \in (\delta_d, \delta_n)$ .

The renegotiation threshold is

$$\delta_n = \frac{z_1}{z_1 - 1} \left( \frac{c_B}{r} - (1 - q)\theta \right) \frac{1}{L + q(U_i - L)}.$$
(16)

The renegotiation policy (16) reduces to (11) when q = 0. Suppose there is no CDS market,  $\theta = 0$ . Because shareholders risk failure in a private workout, they have reduced incentives to default strategically when the renegotiation cost increases.<sup>32</sup>

More importantly, the presence of renegotiation costs weakens the commitment effect of the CDS trading in reducing strategic defaults. To see this, note that  $\frac{\partial \delta_n}{\partial \theta} = -\frac{z_1}{z_1-1} \frac{1-q}{L+q(U_i-L)}$  and  $\frac{\partial}{\partial q} |\frac{\partial \delta_n}{\partial \theta}| < 0$ . Intuitively, the equity pays the additional debt service induced by the debt's credit protection,  $\theta$ , with probability 1-q. Therefore, given a fixed CDS position, a higher renegotiation cost implies

<sup>&</sup>lt;sup>31</sup>Alternatively, we can follow Mella-Barral and Perraudin (1997) in assuming a fixed cost k per unit of time during renegotiation. This formulation implies that the dividend is  $(1 - \tau)(\delta - (r - \mu - i)L\delta - k)$  for  $\delta < \delta_n$  and that the equity has increased incentives to go bankrupt because they absorb extra losses k. However, as the renegotiation cost directly affects the bankruptcy decision, its effects on CDS hedging is less clear.

<sup>&</sup>lt;sup>32</sup>This result holds given a positive CDS position  $\theta$  that is not "too large". Formally, we can show  $\frac{\partial \delta_n}{\partial q} < 0$  for the value-maximizing firms, in which case the bankruptcy threshold is  $\delta_d = \frac{z_1}{z_1-1}\theta \frac{1}{U_i-L}$ . Differentiating (16) with respect to q, we have  $\frac{\partial \delta_n}{\partial q} = \frac{z_1}{z_1-1} \frac{U_i \theta - (U_i - L)c_B/r}{(L+q(U_i-L))^2}$ . Hence  $\frac{\partial \delta_n}{\partial q} \leq 0$  if and only if  $\frac{c_B}{r} \left(1 - \frac{L}{U_i}\right) \geq \theta$ , which holds if  $\delta_d \leq \delta_n$ . Hence,  $\theta$  being not "too large" means the creditor does not choose a  $\theta$  such that  $\delta_d > \delta_n$ . In fact, the creditor at most hedges up to a  $\bar{\theta}$  such that  $\delta_d(\bar{\theta}) = \delta_n(\bar{\theta})$ : Any  $\theta > \bar{\theta}$  is not optimal because the renegotiation does not occur on the equilibrium path and hegding no longer provide strategic benefits.

Table 2: The effect of variations in the renegotiation cost on the financial and investment policies of CDS and non-CDS firms to renegotiation costs. Panel A reports the full model and Panel B shuts down the CDS market. The common parameters are r = 5%,  $\mu = 1\%$ , i = 2.5%,  $\phi = 12.5$ ,  $\sigma = 25\%$ ,  $\tau = 20\%$ , and  $\alpha = 35$ . The initial cash flows is  $\delta_0 = 10$ . We compute the loss as follows. For each level of q, we find the optimal capital structure and compute a hypothetical firm value using "assets plus tax shields minus bankruptcy costs". Such a value does not directly depend on q. Then, we compute the "loss" as  $(1 - V(\delta_0)/\text{hypothetical value}) \times 100\%$ .

q	$V(\delta_0)$	$E(\delta_0)$	$D(\delta_0)$	$ML(\delta_0)$	$\delta_n$	$\delta_d$	loss	$\frac{\theta^*}{c_B^*/r}$	$AC(\delta_0)$	$\frac{\Delta \text{Asset}}{A_0(\delta_0)}$	
Panel A: Non-CDS firms											
0%	351.00	221.00	130.00	37.04%	10.00	-	-				
10%	342.62	226.17	116.45	33.99%	5.57	-	1.63%				
25%	341.18	226.21	114.97	33.70%	4.07	-	1.96%				
50%	342.50	213.85	128.65	37.56%	3.27	-	2.35%				
Panel B: CDS firms											
0%	354.07	148.20	205.87	58.15%	5.39	2.77	-	51.6%	0.376%	-2.18%	
10%	353.46	149.57	203.89	57.68%	4.89	2.69	0.48%	51.4%	0.383%	-1.81%	
25%	353.77	147.84	205.93	58.21%	4.57	2.61	0.72%	49.0%	0.371%	-1.69%	
50%	355.14	140.29	214.85	60.49%	4.36	2.49	1.11%	44.6%	0.340%	-1.51%	

that the equity pays less to the debt in the private workout because the expected interest payment is  $(1-q)\theta$ . This effect weakens the strategic benefit of the creditor's CDS position.

Panel A of Table 2 summarizes the effect of the renegotiation cost on non-CDS firms under the optimal capital structure. As argued above, the renegotiation threshold is decreasing in the renegotiation cost. We also observe a non-monotonicity of the optimal leverage and the debt value in the renegotiation cost. As q increases, there are two effects: first, equity holders finance the firm with less debt ex-ante because they anticipate that high leverage may trigger renegotiation more frequently. Second, the increase in the negotiation friction deters strategic default and makes the debt value more sensitive to the contractual coupon. As the ex-ante optimal leverage internalizes the debt value, it becomes higher than the q = 0 case when the strategic default incentive is low, i.e., when the renegotiation cost is sufficiently high.

Panel B of Table 2 is for CDS firms. Consider an increase in the renegotiation cost. An important observation is that the optimal hedge ratio will decrease due to the reduced strategic benefit of CDS trading. As creditors become less tough in bargaining, equity holders delay formal bankruptcy. Whereas equity holders deleverage to avoid the risk of negotiation failure for low renegotiation costs, the reduced bankruptcy cost for high renegotiation costs is sufficiently large to incentivize equity holders to lever up.

In addition, the renegotiation frictions destroy values. In both panels, we report the loss in value

due to costly on-path renegotiation in the "loss" column. The losses could be significant for both types of firms but smaller for CDS firms. The reason is that the increase in the renegotiation cost not only reduces the default risk but also mitigates the debt overhang caused by the empty creditor channel. The last two columns of panel B quantify the under-investment problem. In particular, the agency cost of CDS decreases with the renegotiation friction in general, and the percentage reduction in asset value relative to the asset of non-CDS firms decreases monotonically.<sup>33</sup>

### 5.3 Socially Optimal Credit Protection

This section analyzes the level of credit protection that maximizes firm value. Formally, for each coupon rate, we define the socially optimal CDS position as

$$\theta^s(c_B) = \underset{\theta \in [0,\infty)}{\arg\max} E(\delta_0; T(\theta, c_B), c_B) + B(\delta_0; T(\theta, c_B), c_B)),$$

given the equity's ex-post decisions. Note that we can always decompose the firm value as

$$E(\delta) + B(\delta) = E(\delta) + \underbrace{(b(\delta) - P(\delta))}_{\text{debt's expected payoff}} + \underbrace{(P(\delta) - C(\delta))}_{\text{seller's payoff}}.$$

The expression shows that the socially optimal CDS position maximizes the sum of the payoffs of all claims holders and the CDS seller. At  $\delta_0$ , competitive pricing implies that the CDS seller breaks even, and the firm's trade-off of the socially optimal CDS position is as follows: On the one hand, an increase in  $\theta$  improves the ex-ante firm value because the credit protection strengthens the creditor's bargaining position. The benefit is reflected in the increased in  $b(\delta)$ . On the other hand, a higher  $\theta$  reduces the equity value and increases the CDS price due to an increased bankruptcy change and more severe under-investment. Moreover, the equity value reduces because shareholders can take less advantage of the strategic default option.

Compared to the socially optimal CDS position, the creditor does not internalize the reduction in the equity value when choosing the privately optimal credit protection. Consequently, the creditor over-hedges against credit risk and the excessively high bankruptcy likelihood yields inefficiency.

**Proposition 6.** The debt holders always over-hedge against credit risk  $\theta^*(c_B) > \theta^s(c_B)$ .

Figure 5 depicts the socially and privately optimal credit protection for both the equity-

<sup>&</sup>lt;sup>33</sup>From q = 0% to q = 10%, the agency cost of CDS increases because of the scaling in the firm value. Indeed, the difference  $V_{FB}(\delta_0) - V_{CDS}(\delta_0)$  decreases monotonically with the renegotiation cost.



Figure 5: The socially optimal and private optimal CDS position. The parameters are r = 5%,  $\mu = 1\%$ ,  $\sigma = 25\%$ , i = 2.5%,  $\phi = 12.5$ ,  $\tau = 20\%$ ,  $\alpha = 35\%$ , and initial cash flows  $\delta_0 = 10$ . We plot the values under the optimal coupon,  $c_B^* = 12.7$ , of the baseline CDS firm.

maximizing and value-maximizing firms. The figure shows that even without the debt-overhang effect, the creditor still over-insures because the creditors have no incentive to internalize the equity value holds regardless of the investment option. Furthermore, the difference in the slopes of the increasing part of the firm value and debt value again reveals the wealth transfer effect of CDS contracts. The steeper debt value reflects that debt holders benefit disproportionately more than the firm when the credit protection reduces the limited commitment frictions in the firm.

#### 5.4 Predicting Investment

Our model provides a new perspective on dynamic investment: CDS firms, compared to non-CDS firms, have larger debt-financed investment at debt issuance times and lower (internal) equity-financed investment once the debt is in place. Specifically, suppose at time 0, the cost of setting up the assets is K and the entrepreneur finances the initial investment cost with debt. One can conceive situations in which  $B_0(\delta_0) < K$  but  $B_{CDS}(\delta_0) \geq K$ . Hence the positive effect of CDS trading on the debt capacity allows the firm to break the initial financial constraint. However, in the paths where the CDS firms are not actively issuing debt, debt overhang implies a lower investment rate.

Several empirical studies have investigated the real impacts of credit derivatives recently. All of these works motivate their test hypotheses from the empty creditor problem. None of them explicitly links the empirical findings to the debt overhang problem. Colonello, Efing, and Zucchi (2016) document that CDS firms with strong shareholders cut investment by 7% (as measured by capital expenditures over lagged property, plant, and equipment (PPE)).<sup>34</sup> This observation is consistent with the debt overhang after the initiation of CDS trading.

Additionally, Guest, Karapatsas, Petmezas, and Travlos (2017) investigate how CDS affects corporate acquisitions and find that CDS firms have a 4.99% higher acquisition propensity than non-CDS firms do. The result is consistent with the hypothesis of increased debt capacity. Batta and Yu (2017) investigate firms' investment in a more detailed way. Post-CDS introduction, they find that (i) asset growth declines by approximately 2.1%, with a significant decrease in net investment and cash paid for acquisitions, and (ii) net debt issuance declines by an average of 1.1%.<sup>35</sup> Interestingly, they also estimate a positive effect of CDS trading on investment, cash paid for acquisitions and debt issuance at the CDS introduction years. The last finding is consistent with the hypothesis that debt-financed acquisitions can be executed more efficiently with expanded debt capacity due to the initiation of CDS trading.

Our model provides a consistent explanation of the empirical findings: firms with CDS trades have increased investment at any debt issuance time or refinancing point, and the debt overhang effect implies reduced investment and asset growth. In spirit, this logic is in line with the distinction between the *true dynamics* and *refinancing points* discussed by Strebelueav (2007). Consider the panel regression model used in some of the above empirical studies

$$I_{it} = \beta_0 + \beta_1 \cdot CDStrading_{it} + \gamma X_{it} + \epsilon_{it},$$

where  $I_{it}$  measures investment,  $CDStrading_{it} = 1$  if the firm has CDS traded at time t and 0 otherwise, and  $X_{it}$  is the control of the firm's characteristics. On the one hand, the ex-ante positive effect implies that  $\beta_1 > 0$  when we perform the regression analysis using data at the debt issuance times. On the other hand, a panel regression using all the available data is likely to incorporate a significant amount of ex-post negative effect because of infrequent debt issuance, and thus  $\beta_1 < 0$ .

<sup>&</sup>lt;sup>34</sup>Their paper provides a static model without corporate taxes that generates under-investment with CDS trading, and their mechanism is different from ours. Their firm uses both equity and debt in financing an investment project, that is,  $E(\delta_0) + D(\delta_0) \ge K$  needs to be satisfied. They prove that the firm value will decrease with traded CDS and, hence that under-investment occurs as the financing condition could be violated. We argue that the financing condition is likely to be satisfied with the usual corporate tax rate and the optimal choice of capital structure.

<sup>&</sup>lt;sup>35</sup>Our model assumes a one-shot capital structure choice and thus does not capture the declines in debt issuance. We believe that the extended model that incorporates dynamic capital structure decision is able to explain the result. See footnote 30. Also, recall that in Table 1, CDS trading transfers values from the equity holders to the debt holders. Facing future debt issuance opportunities, the equity holders have reduced incentives to issue new debts as they expect dilution caused by CDS trading.

 $\beta_1$  seems to have cross-sectional variation.

Several additional results can be tested by including an interaction term  $CDStrading_{it} \times Z_{it}$ . The independent variable  $Z_{it}$  can be a variable that controls for the CEO or institutional shareholdings that measure the equity's bargaining power; or the number of creditors and the number of bond issues that measure the renegotiation frictions (Davydenko and Strebulaev, 2007) for testing the implication in Section 5.1 and 5.2. Our comparative statics analysis (Section 4.2) suggests that the adverse effect of CDS trading on ex-post investment is more pronounced for firms with greater cash-flow risk and more tangible assets.

Overall, our dynamic model implies that the ex-ante positive effect on investment applies to times of debt issuance or refinancing, and the ex-post negative effect applies once the debt is in place. Empirical designs and hypothesis development should take the timing of CDS-implied effects into account.

#### 5.5 Connection to Securitization

Following common practice, our model assumes that the CDS seller insures creditors (the CDS buyer) against bankruptcy by compensating them  $\theta$  upon default. One may interpret the compensation  $\theta$  as the net payment resulting from creditors putting the defaulted loan (valued at  $L\delta$ ) to the seller in exchange for a gross payment  $L\delta + \theta$ . Put together, creditors' holdings of the *debt* and *the put option (the CDS) written on the underlying debt* create the empty creditor problem by separating creditors' cash flow rights from their liquidation right. As a consequence, creditors' reservation value increases from  $L\delta$  to  $L\delta + \theta$  and, therefore, they demand greater interest repayments in debt renegotiations. Anticipating this phenomenon, shareholders postpone debt renegotiations and under-invest in value-enhancing projects.

It is worth noting that these results are not necessarily limited to CDS protection and may be generalized to other credit risk transfer mechanisms such as securitization. Consider, for example, a bank that originates a loan and then sells the loan off its balance sheet through securitiziation. To that end, the bank establishes a special purpose vehicle (SPV) and issues loan-backed securities to secondary market investors. Standard risk retention rules require the bank to have "skin in the game." For ease of exposition, we restrict our attention to a simple capital structure of (risky) debt and equity of the SPV and assume the bank retains an equity interest (a first loss position).<sup>36</sup> As

 $<sup>^{36}</sup>$ Hartman-Glaser, Piskorski, and Tchistyi (2012) show that the optimal securitization contract can be closely approximated by a first loss position.

the SPV is a passive entity, the liquidation right of the loan remains with its sponsor: the bank.

As the bank essentially holds a levered position of the loan, it is entitled to the residual loan payment after servicing the SPV's debt investors. Unlike the full ownership case where the bank absorbs all losses made to the loan, with a levered position the bank's loss on the loan is truncated at zero due to limited liability. This provides an insurance against downside risk and allows the bank to improve its bargain position and demand greater interest repayments in debt renegotiations. In particular, the bank will not accept any strategic interest repayment less than the debt it owes to the SPV's debt investors.

Finally, if the bank structures the SPV such that it carries a debt obligation of  $L\delta + \theta$ , then securitization generates the same real and financial implications as those derive in the model with CDS protection.<sup>37</sup> In fact, our discussion at the beginning of the subsection implies that a CDS protected bank essentially holds a portfolio of a loan and a put option (with a strike price  $L\delta + \theta$ ) written on the underlying loan. Alternatively, through securitization and leveraging the SPV to  $L\delta + \theta$ , the bank holds an equity position on the underlying loan, which has a natural interpretation as a call option (with the same strike price). The put-call parity yields that CDS protection and securitization may result in the same risk exposure and bargain positions for the bank. It follows that the bank's borrower (the firm) makes the same real and financial decisions.

## 6 Conclusion

We analyze both the financial and real implications of the introduction of a CDS market. We provide a tractable model with endogenous CDS positions, dynamic investment opportunities, and the optimal capital structure in the spirit of Leland (1994). We show that with renegotiable debts, the strategic benefit of CDS trading expands the debt capacity and increases the optimal leverage. Moreover, the credit spread, which measures the cost of corporate debt, decreases as a response to the inception of CDS trading when the equity's ex-post bankruptcy decision is taken into account. This positive effect allows ex-ante financially constrained firms to undertake a larger set of positive NPV projects, consistent with the prediction of the static model provided by Bolton and Oehmke (2001). We also find that, for a wide range of parameter constellations, having an active CDS market tends to increase firm value.

Our dynamic analysis uncovers a novel and negative real impact of CDS trading stemming

<sup>&</sup>lt;sup>37</sup>If the bank would assume the full ownership of the loan (that is, without securitizing the loan), it will accept only a debt renegotiation offer greater than its reservation value  $L\delta$ , corresponding to the non CDS case ( $\theta = 0$ ).

from the empty creditor channel. The empty creditor problem implies that the equity holders face tougher debt holders in the future when the firm's fundamental is weak; and the incentive for the equity to declare bankruptcy increases with the notional amount of the CDS held by debt holders. This drives debt overhang and reduces the equity's incentive to invest in the future. We quantify the CDS-induced debt overhang cost using the gain in firm value when the equity holders could commit to an efficient investment policy, and the difference in asset value for CDS and non-CDS firms. Our calibration shows that both estimates are non-trivial. We further show that the debt overhang implied by the empty creditor channel becomes more severe if equity holders have higher bargaining power or face lower renegotiation costs, the debt holders hedge excessively, and when the firm has a lower business risk or liquidation value.

In sum, our model takes the view that the introduction of CDS trading is beneficial for firms at the expense of increased under-investment. In evaluating the effect of a policy that bans naked CDS positions or any regulatory changes that concern reorganization procedures, policymakers might want to balance ex-ante financial benefits and ex-post real costs. We believe our framework is useful for further studies that explore the dynamic interaction of the CDS market and the financial and real policies of firms. Our model can be structurally estimated, and such an exercise is likely to provide further insights.

## Appendix A

Appendix for Section 3.1 (General contingent claims and the valuation of CDS contracts). Consider a contingent claim  $C(\delta)$  that pays  $\tilde{p}$  continuously and  $p_D$  at bankruptcy. The value  $C(\delta)$  solves the ODE

$$r\mathcal{C}(\delta) = \begin{cases} (\mu+i)\delta\mathcal{C}'(\delta) + \frac{1}{2}\sigma^2\delta^2\mathcal{C}''(\delta) & \text{if } \delta \ge \delta_i; \\ \mu\delta\mathcal{C}'(\delta) + \frac{1}{2}\sigma^2\delta^2\mathcal{C}''(\delta) & \text{if } \delta_d < \delta < \delta_i \end{cases}$$

The general solution is given by

$$\mathcal{C}(\delta) = \begin{cases} \frac{\tilde{p}}{r} + K_1 \delta^{z_1} & \text{if } \delta \ge \delta_i; \\ \frac{\tilde{p}}{r} + K_2 \delta^{a_0} + K_3 \delta^{z_0} & \text{if } \delta_d < \delta < \delta_i. \end{cases}$$
(17)

We pin down the coefficients  $K_1$ ,  $K_2$ , and  $K_3$  using the boundary conditions: First, value-matching and smooth-pasting at  $\delta_i$  imply  $K_1 = K_2 \delta_i^{a_0-z_1} + K_3 \delta_i^{z_0-z_1}$  and  $K_2 = \frac{z_0-z_1}{z_1-a_0} \delta_i^{z_0-a_0} K_3$ . Observe that  $\tilde{p}$  and  $p_D$  only affect these two equations through their effect on  $K_3$ . And second, value-matching  $\mathcal{C}(\delta_d) = p_D$  at bankruptcy gives  $K_3 = \frac{p_D - \tilde{p}/r}{\left(\frac{z_0-z_1}{z_1-a_0} \delta_i^{z_0-a_0} \delta_d^{a_0} + \delta_d^{z_0}\right)}$ . Hence, given  $\delta_i$  and  $\delta_d$ ,  $K_3$  is linear in  $\tilde{p}$  and  $p_D$ . We obtain the expected value of credit protection  $C(\delta)$  by setting  $p_D = \theta$  and  $\tilde{p} = 0$ . Therefore,

We obtain the expected value of credit protection  $C(\delta)$  by setting  $p_D = \theta$  and  $\tilde{p} = 0$ . Therefore,  $C_3 = \frac{\theta}{\left(\frac{z_0 - z_1}{z_1 - a_0}\delta_i^{z_0 - a_0}\delta_d^{a_0} + \delta_d^{z_0}\right)}, C_2 = \frac{z_0 - z_1}{z_1 - a_0}\delta_i^{z_0 - a_0}C_3$ , and  $C_1 = C_2\delta_i^{a_0 - z_1} + C_3\delta_i^{z_0 - z_1}$ . Then for  $\delta_d < \delta < \delta_i$ ,  $C(\delta) = \theta \cdot \left(\frac{z_0 - z_1}{z_1 - a_0}\delta_i^{z_0 - a_0}\delta_a^{a_0} + \delta_d^{z_0}\right) / \left(\frac{z_0 - z_1}{z_1 - a_0}\delta_d^{z_0 - a_0}\delta_d^{a_0} + \delta_d^{z_0}\right)$ ; and for  $\delta \ge \delta_i$ ,  $C(\delta) = \theta \cdot \left(\frac{z_0 - z_1}{z_1 - a_0}\delta_i^{z_0 - z_1}\delta_i^{z_1 - z_1}$ 

Similarly, for the premium leg, we have  $P_3 = \frac{-p/r}{\left(\frac{z_0-z_1}{z_1-a_0}\delta_i^{z_0-a_0}\delta_d^{a_0}+\delta_d^{z_0}\right)}$ ,  $P_2 = \frac{z_0-z_1}{z_1-a_0}\delta_i^{z_0-a_0}P_3$ , and  $P_1 = P_2\delta_i^{a_0-z_1} + P_3\delta_i^{z_0-z_1}$  by setting  $\tilde{p} = p$  and  $p_D = 0$ . Hence, for  $\delta_d < \delta < \delta_i$ ,

$$P(\delta) = \frac{p}{r} \cdot \left( 1 - \left( \frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta^{a_0} + \delta^{z_0} \right) / \left( \frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta_d^{a_0} + \delta_d^{z_0} \right) \right);$$

and for  $\delta \geq \delta_i$ ,  $P(\delta) = \frac{p}{r} \cdot \left(1 - \left(\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - z_1} \delta^{z_1} + \delta_i^{z_0 - z_1} \delta^{z_1}\right) / \left(\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta_d^{a_0} + \delta_d^{z_0}\right)\right)$ . The expressions suggest that we can define

$$P_d^i(\delta) \equiv \frac{\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - z_1} \delta^{z_1} + \delta_i^{z_0 - z_1} \delta^{z_1}}{\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta_d^{a_0} + \delta_d^{z_0}}, \text{ and } P_d^0(\delta) \equiv \frac{\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta^{a_0} + \delta^{z_0}}{\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta_d^{a_0} + \delta_d^{z_0}}$$

as the bankruptcy probability when the firm is investing and not investing respectively. Note that (i) continuity implies  $\lim_{\delta \uparrow \delta_i} P_d^0(\delta) = \lim_{\delta \downarrow \delta_i} P_d^i(\delta)$ ; (ii)  $P_d^0(\delta_d) = 1$ ; and (iii)  $P_d^0(\delta) \to \left(\frac{\delta}{\delta_d}\right)^{z_0}$  as  $\delta_i \to \infty$ .

Finally, using the competitive pricing condition (3), we have the CDS premium p satisfying the following

equations:

$$\frac{p}{r}\left(1 - P_d^i(\delta_0)\right) = \theta \cdot P_d^i(\delta_0)$$

for  $\delta_0 \geq \delta_i$ ; and

$$\frac{p}{r}\left(1 - P_d^0(\delta_0)\right) = \theta \cdot P_d^0(\delta_0)$$

for  $\delta_d < \delta_0 < \delta_i$ .

Appendix for Section 3.2.1 and 3.2.2 (The valuation of debt and equity). The ODE for the debt payoff (4) can be written more explicitly as

$$rb(\delta) = \begin{cases} c_B + (\mu + i)\delta b'(\delta) + \frac{1}{2}\sigma^2\delta^2 b''(\delta) & \text{if } \delta > \delta_n; \\ s(\delta) + (\mu + i)\delta b'(\delta) + \frac{1}{2}\sigma^2\delta^2 b''(\delta) & \text{if } \delta_i \le \delta \le \delta_n; \\ s(\delta) + \mu\delta b'(\delta) + \frac{1}{2}\sigma^2\delta^2 b''(\delta) & \text{if } \delta_d < \delta < \delta_i. \end{cases}$$

As  $b(\delta) = R(\delta) = \theta + L\delta$  for  $\delta \in (\delta_d, \delta_n]$ , the general solution is

$$b(\delta) = \begin{cases} \frac{c_B}{r} + B_1 \delta^{z_1} & \text{if } \delta > \delta_n; \\ \theta + L\delta & \text{if } \delta_d < \delta \le \delta_n \end{cases}$$

Note that  $b(\delta)$  automatically satisfies value-matching and smooth-pasting at  $\delta_i$ ; and value-matching at  $\delta_n$  yields  $B_1 = \left(\theta + L\delta_n - \frac{c_B}{r}\right)\delta_n^{-z_1}$ . This gives the debt payoff (6).

To derive the solution for the equity value (7), we use  $V(\delta) = E(\delta) + B(\delta)$ . According to the Trade-off theory, the firm value can be written as

$$V(\delta) = \underbrace{A(\delta)}_{\text{unlevered asset}} + \underbrace{\tau \left(B(\delta) - L(\delta)\right)}_{\text{tax shield}} - \underbrace{BC(\delta)}_{\text{bankruptcy cost}},$$

where  $L(\delta)$  is the liquidation value of the firm. So  $B(\delta) - L(\delta)$  captures the present value of interest payments before bankruptcy. The claim  $L(\delta)$  and  $BC(\delta)$  have general solutions (17) with  $\tilde{p} = 0$  and values at bankruptcy  $L(\delta_d) = L\delta_d$  and  $BC(\delta_d) = \alpha(1-\tau)U_0\delta_d$ . Thus, for  $\delta \ge \delta_i$ ,  $L(\delta) = L\delta_d \cdot P_d^i(\delta)$  and  $BC(\delta) = \alpha(1-\tau)U_0\delta_d \cdot P_d^i(\delta)$ ; and for  $\delta_d < \delta < \delta_i$ ,  $L(\delta) = L\delta_d \cdot P_d^0(\delta)$  and  $BC(\delta) = \alpha(1-\tau)U_0\delta_d \cdot P_d^0(\delta)$ . Define a claim  $A^d(\delta)$  that pays  $(1-\tau)U_0\delta_d$  at bankruptcy and zero before bankruptcy, then  $L(\delta) = A^d(\delta) - BC(\delta)$ . By rearranging terms and simplifying, we can express the equity value as

$$E(\delta) = \underbrace{A(\delta) - (1 - \tau)b(\delta)}_{\text{dividend+invest. option}} + \underbrace{(1 - \tau)\left(C(\delta) + L(\delta) - \frac{A^d(\delta)}{1 - \tau}\right)}_{\text{default option}}.$$

Now for the unlevered asset value, the general solution is

$$A(\delta) = \begin{cases} \frac{(1-\tau)(1-\tilde{\phi}i)}{r-(\mu+i)}\delta + A_1\delta^{z_1} & \text{if } \delta \ge \delta_i;\\ \frac{1-\tau}{r-\mu}\delta + A_2\delta^{a_0} + A_3\delta^{z_0} & \text{if } \delta_d < \delta < \delta_i \end{cases}$$

To determine the coefficients  $A_1$ ,  $A_2$ , and  $A_3$ , we impose three boundary conditions. First, value-matching at bankruptcy:  $A(\delta_d) = U_0 \delta_d$ . This yields  $\frac{1-\tau}{r-\mu} \delta_d + A_2 \delta_d^{a_0} + A_3 \delta_d^{z_0} = \frac{1-\tau}{r-\mu} \delta_d$ . Second, value-matching and smooth-pasting at investment:  $\frac{(1-\tau)(1-\tilde{\phi}i)}{r-(\mu+i)} \delta_i + A_1 \delta_i^{z_1} = (1-\tau) \frac{\delta_i}{r-\mu} + A_2 \delta_i^{a_0} + A_3 \delta_i^{z_0}$  and  $\frac{(1-\tau)(1-\tilde{\phi}i)}{r-(\mu+i)} \delta_i + z_1 A_1 \delta_i^{z_1} = \frac{1-\tau}{r-\mu} \delta_i + a_0 A_2 \delta_i^{a_0} + z_0 A_3 \delta_i^{z_0}$ . With some algebra, we can show that

$$A_2 = \frac{(1-z_1)(1-\tau)(U_i-U_0)\delta_i}{(a_0-z_1)\delta_i^{a_0} - (z_0-z_1)\left(\frac{\delta_i}{\delta_d}\right)^{z_0}\delta_d^{a_0}}$$

and using the value-matching at bankruptcy, we have

$$A(\delta) = \frac{1-\tau}{r-\mu}\delta + A_2\left(\delta^{a_0} - \left(\frac{\delta}{\delta_d}\right)^{z_0}\delta^{a_0}_d\right) = \frac{1-\tau}{r-\mu}\delta + (1-\tau)(U_i - U_0)\delta_i \cdot \Gamma_0(\delta, \delta_d),$$

for  $\delta_d < \delta < \delta_i$ . Here,  $\Gamma_0(\delta, \delta_d) \equiv \frac{(1-z_1)\delta^{a_0} - (1-z_1)\left(\frac{\delta}{\delta_d}\right)^{z_0}\delta^{a_0}_d}{(a_0-z_1)\delta^{a_0}_i - (z_0-z_1)\left(\frac{\delta_i}{\delta_d}\right)^{z_0}\delta^{a_0}_d}$ . Similarly, for  $\delta \ge \delta_i$ ,

$$A(\delta) = \frac{(1-\tau)(1-\tilde{\phi}i)}{r-(\mu+i)}\delta + \left(A_2\left(\delta_i^{a_0} - \left(\frac{\delta_i}{\delta_d}\right)^{z_0}\delta_d^{a_0}\right) - (1-\tau)(U_i - U_0)\delta_i\right)\left(\frac{\delta}{\delta_i}\right)^{z_1}$$
$$= \frac{(1-\tau)(1-\tilde{\phi}i)}{r-(\mu+i)}\delta + (1-\tau)(U_i - U_0)\delta_i \cdot \Gamma_i(\delta_i, \delta_d)\left(\frac{\delta}{\delta_i}\right)^{z_1},$$

where  $\Gamma_i(\delta_i, \delta_d) \equiv \frac{(1-a_0)\delta_i^{a_0} - (1-z_0)\left(\frac{\delta_i}{\delta_d}\right)^{z_0}\delta_d^{a_0}}{(a_0-z_1)\delta_i^{a_0} - (z_0-z_1)\left(\frac{\delta_i}{\delta_d}\right)^{z_0}\delta_d^{a_0}}$ . Substituting all the solved components, we obtain the equity value (8), (9), and (10).

Endogenous thresholds and the proof of Proposition 1. For the bankruptcy threshold, the smoothpasting condition  $E'(\delta_d) = 0$  is equivalent to  $E'(\delta_d)\delta_d = 0$ . Therefore, for a given  $\delta_i$ ,  $\delta_d$  solves

$$(1-\tau)\left(U_0-L\right)\delta_d + (1-\tau)\left(\theta + L\delta_d - \frac{\delta_d}{r-\mu}\right)\frac{\partial P_d^0(\delta_d)}{\partial\delta}\delta_d + (1-\tau)\Pi\delta_i\Gamma_0'(\delta_d,\delta_d) = 0.$$
 (18)

In (18),  $\frac{\partial P_d^0(\delta_d)}{\partial \delta} \delta_d = \frac{a_0 \cdot \frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta_d^{a_0} + z_0 \delta_d^{z_0}}{\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta_d^{z_0 - a_0} \delta_d^{a_0} + \delta_d^{z_0}} \text{ and } \Gamma_0'(\delta_d, \delta_d) = \frac{(a_0 - z_0)(1 - z_1) \delta_d^{a_0}}{(a_0 - z_1) \delta_i^{a_0} - (z_0 - z_1) \left(\frac{\delta_i}{\delta_d}\right)^{z_0} \delta_d^{a_0}}.$  For the investment threshold, the optimality condition  $E'(\delta_i) = \phi$  is equivalent to  $E'(\delta_i) \delta_i = \phi \delta_i$ . Therefore, for a given  $\delta_d$ ,  $\delta_i$  solves

$$(1-\tau)\left(U_i-L\right)\delta_i + (1-\tau)\left(\theta + L\delta_d - \frac{\delta_d}{r-\mu}\right)\frac{\partial P_d^i(\delta_i)}{\partial\delta}\delta_i + z_1(1-\tau)\Pi\delta_i \cdot \Gamma_i(\delta_i,\delta_d) = \phi\delta_i.$$
 (19)

In (19),  $\frac{\partial P_d^i(\delta_i)}{\partial \delta} \delta_i = \frac{z_1 \frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0} + z_1 \delta_i^{z_0}}{\frac{z_0 - z_1}{z_1 - a_0} \delta_i^{z_0 - a_0} \delta_d^{a_0} + \delta_d^{z_0}}$ . Note that in deriving  $E'(\delta_i)$ , we use (8). We obtain the same condition if we use (9). This is because the equity value satisfies the smooth-pasting condition at the investment threshold. The solution to the system of nonlinear equations (18) and (19) is the optimal bankruptcy and investment thresholds.

To obtain more information, we write the system (18) and (19) as follows. Noting that both  $\Gamma'_0(\delta_d, \delta_d)$ and  $\Gamma_i(\delta_i, \delta_d)$  depends only on the ratio  $\delta_i/\delta_d$ , we denote  $y = \frac{\delta_i}{\delta_d}$  and express the two as  $\Gamma_0(y)$  and  $\Gamma_i(y)$ respectively. Moreover,

$$\frac{\frac{\partial P_d^i(\delta_i)}{\partial \delta}\delta_i}{\frac{\partial P_d^0(\delta_d)}{\partial \delta}\delta_d} = \frac{z_1\left(\frac{z_0-z_1}{z_1-a_0}+1\right)\left(\frac{\delta_i}{\delta_d}\right)^{z_0+a_0}}{a_0\left(\frac{z_0-z_1}{z_1-a_0}\right)\left(\frac{\delta_i}{\delta_d}\right)^{z_0}+z_0\left(\frac{\delta_i}{\delta_d}\right)^{a_0}} \equiv G_1(y); \text{ and } \frac{\partial P_d^0(\delta_d)}{\partial \delta}\delta_d = \frac{a_0\frac{z_0-z_1}{z_1-a_0}\left(\frac{\delta_i}{\delta_d}\right)^{z_0}+z_0\left(\frac{\delta_i}{\delta_d}\right)^{a_0}}{\frac{z_0-z_1}{z_1-a_0}\left(\frac{\delta_i}{\delta_d}\right)^{z_0}+\left(\frac{\delta_i}{\delta_d}\right)^{a_0}} \equiv G_2(y)$$

Then using (18) to eliminate  $\left(\theta + L\delta_d - \frac{\delta_d}{r-\mu}\right)$  in (19), we obtain

$$(U_i - L) - (\Pi \cdot \Gamma_0(y) + (U_0 - L)) G(y) + z_1 \Pi \cdot \Gamma_i(y) = \tilde{\phi},$$
(20)

and from equation (18), we have

$$\frac{\theta}{\delta_d} = \frac{\left(G_2(y) - 1\right)\left(U_0 - L\right) - \Pi \cdot y \cdot \Gamma_0(y)}{G_2(y)},\tag{21}$$

respectively. The system implies that given a fixed set of parameters, the nonlinear equation (20) first determines  $y = \delta_i/\delta_d$  regardless of  $\theta$ . Next, the bankruptcy boundary  $\delta_d$  can be solved from equation (21), given the solution y and the CDS position  $\theta$ . From equation (21), we also know  $\theta/\delta_d > 0$  because  $\Gamma_0(y) > 0$  and  $G_2(y) < 0$ . It follows that  $\delta_d$  is increasing in  $\theta$ ; and through the fixed ratio y,  $\delta_i$  is also increasing in  $\theta$ .

**Proof of Proposition 2.** Within the renegotiation region  $0 < \delta \leq \delta_n$ , the equity value is given by

$$E(\delta) = \begin{cases} (1-\tau) \left(\frac{1-\tilde{\phi}i}{r-(\mu+i)}\delta - L\delta\right) - \frac{a_0-1}{a_0-z_1}\tilde{\Pi}\delta_i \left(\frac{\delta}{\delta_i}\right)^{z_1} & \text{if } \delta_i \le \delta \le \delta_n; \\ (1-\tau) \left(\frac{\delta}{r-\mu} - L\delta\right) + \frac{1-z_1}{a_0-z_1}\tilde{\Pi}\delta_i \left(\frac{\delta}{\delta_i}\right)^{a_0} & \text{if } 0 < \delta < \delta_i, \end{cases}$$

where  $\tilde{\Pi} = (1 - \tau) \left( \frac{1 - \tilde{\phi}i}{r - (\mu + i)} - \frac{1}{r - \mu} \right)$  is the increment of the present value of cash flows per unit of  $\delta$  from investment. The second terms on the right-hand side of the expression capture the value of the investment options. The equity value can be obtained by letting  $\theta = 0$  and  $\delta_d = 0$  in (8), (9), and (10). Note that for  $\delta \geq \delta_i$ ,

$$E'(\delta) = (1 - \tau) (U_i - L) + (-z_1) \frac{a_0 - 1}{a_0 - z_1} \tilde{\Pi} \left(\frac{\delta_i}{\delta}\right)^{1 - z_1},$$

and hence for  $\delta_i \to 0$ , the second term on the right-hand side converges to 0 for any  $\delta$  because  $z_1$  is the negative root and  $\tilde{\Pi} > 0$ . Therefore, if  $E'(\delta; \delta_i = 0) = (1 - \tau) (U_i - L) > \phi$ , then it is optimal for the equity

holders to invest at all times and  $\delta_i = 0$ .

In fact, the proof of Proposition 1 implies the same result. From there, let  $\theta \to 0$ . Equation (21) implies that  $\delta_d \to 0$  as well for any y > 0. Then the left-hand side of (19) converges to

$$((1-\tau)(U_i - L) - \phi)\,\delta_i + z_1(1-\tau)\Pi \cdot \frac{1-a_0}{a_0 - z_1} \cdot \delta_i = 0.$$

The coefficient in front of  $\delta_i$  in the first term is strictly positive by assumption 1; and the coefficient attached to  $\delta_i$  in the second term is also strictly positive. The only solution for  $\delta_i$  is  $0.\blacksquare$ 

**Proof of Proposition 3.** First, we make a few observations when  $\delta_i \to \delta_d$ : (i)  $P_d^i(\delta) \to \left(\frac{\delta}{\delta_d}\right)^{z_1}$  and  $P_d^0(\delta) \to \left[\frac{z_0-z_1}{z_1-a_0}\left(\frac{\delta}{\delta_d}\right)^{a_0} + \left(\frac{\delta}{\delta_d}\right)^{z_0}\right] / \left[\frac{z_0-z_1}{z_1-a_0} + 1\right]$ . As the length of  $(\delta_d, \delta_i)$  converges to 0, any  $\delta$  in this interval is effectively  $\delta_d$ . So  $P_d^0(\delta) \to 1$ . (ii)  $\Gamma_i(\delta_i, \delta_d) \to -1$ ,  $\Gamma_0'(\delta_d, \delta_d) \to 1-z_1$ ,  $\frac{\partial P_d^0(\delta_d)}{\partial \delta} \delta_d \to z_1$ ,  $\frac{\partial P_d^i(\delta_i)}{\partial \delta} \delta_i \to z_1$ , and similar to the first observation,  $\Gamma_0(\delta, \delta_d) \to \left[(1-z_1)\left(\frac{\delta}{\delta_d}\right)^{a_0} - (1-z_1)\left(\frac{\delta}{\delta_d}\right)^{z_0}\right] / [a_0 - z_0] = 0$  because  $\delta = \delta_d$  on  $(\delta_d, \delta_i)$ . Using these observations, it is easy to check that the equity value (10) on  $(\delta_n, \infty)$  converges to (12); and the equity value (9) on  $(\delta_i, \delta_n)$  converges to (13). Also, the equity value (8) becomes 0. Moreover, given  $\delta_i = \delta_d$ , the smooth-pasting condition for bankruptcy (18) becomes

$$(U_0 - L)\delta_d - (\theta + L\delta_d - U_0\delta_d) z_1 + \Pi\delta_d(1 - z_1) = 0 \Rightarrow \delta_d = \frac{z_1}{z_1 - 1} \frac{1}{U_i - L}\theta.$$

Second, we show that under Assumption 1,  $V'(\delta_d^{FB}; \delta_i^{FB} = \delta_d^{FB}) > \phi$  and hence the investment threshold is a corner solution at  $\delta_d^{FB}$ . To simplify notation, we suppress the FB superscript. By definition,  $V'(\delta) = E'(\delta) + B'(\delta)$ . Since  $E'(\delta_d^{FB}) = 0$ ,

$$V'(\delta_d) \cdot \delta_i = B'(\delta_d) \cdot \delta_i = L\delta_i - \theta \frac{\partial P_d^i(\delta_i)}{\partial \delta} \delta_i$$
$$\Rightarrow V'(\delta_d; \delta_i = \delta_d) \cdot \delta_d = L\delta_d - \theta z_1.$$

because  $\frac{\partial P_d^i(\delta_i)}{\partial \delta} \delta_i \to z_1$  as  $\delta_i \to \delta_d$ . Eliminating  $\theta$  using  $\delta_d = \frac{z_1}{z_1-1} \frac{1}{U_i-L} \theta$ , we have  $V'(\delta_d; \delta_i = \delta_d) \cdot \delta_d = [L + (1 - z_1)(U_i - L)] \delta_d > \phi \delta_d$  if  $L + (1 - z_1)(U_i - L) > \phi$ . But the strict inequality is always satisfied under Assumption 1 because of  $z_1 < 0$  and  $\tau < 1$ , so  $L + (1 - z_1)(U_i - L) > (1 - \tau)(U_i - L) > \phi$ . Therefore,  $\delta_i = \delta_d$  in the first-best.

**Proof of Proposition 4.** We prove the scale invariance in three steps.

**Step 1.** We first show that the value functions are homogeneous of degree one in  $(\delta, c_B, \theta, T)$  where T =

 $(\delta_n, \delta_i, \delta_d)$  is the collection of thresholds. That is, for any constant  $\beta > 0$ ,

$$b(\beta\delta; \beta c_B, \beta\theta, \beta T) = \beta b(\delta; c_B, \theta, T)$$
$$C(\beta\delta; \beta c_B, \beta\theta, \beta T) = \beta C(\delta; c_B, \theta, T)$$
$$E(\beta\delta; \beta c_B, \beta\theta, \beta T) = \beta E(\delta; c_B, \theta, T).$$

To show this, first observe that  $b(\delta; c_B, \theta, T)$  (Equation (6)) is homogeneous of degree one in  $(\delta, c_B, \theta, T)$ . Moreover, it is easy to verify that  $P_d^i(\delta)$  and  $P_d^0(\delta)$  are homogeneous of degree zero in  $(\delta, c_B, \theta, T)$ . Hence,

$$C(\beta\delta;\beta c_B,\beta\theta,\beta T) = \theta P_d^i(\beta\delta;\beta c_B,\beta\theta,\beta T)$$
 and  $C(\beta\delta;\beta c_B,\beta\theta,\beta T) = \theta P_d^0(\beta\delta;\beta c_B,\beta\theta,\beta T)$ 

are homogeneous of degree one in  $(\delta, c_B, \theta, T)$ . Similarly,  $\Gamma_0(\delta)$  and  $\Gamma_i(\delta)$  are homogeneous of degree zero in  $(\delta, c_B, \theta, T)$ . Thus, using equations (8), (9), and (10), we can derive that  $E(\delta; c_B, \theta, T)$  is homogeneous of degree one in all of three regions.

Finally, firm value function  $V(\delta; c_B, \theta, T)$  is also homogeneous of degree one in  $(\delta_0, c_B, \theta, T)$ .

Step 2. Using the homogeneity property and assuming that the collection of thresholds T is linear in  $\delta_0$ , we show that the optimal coupon and CDS protection are linear in the initial cash flow  $\delta_0$ . That is,  $c_B = \gamma \delta_0$  and  $\theta = \eta \delta_0$ , where  $\gamma$  and  $\eta$  are constants independent upon  $\delta_0$ , which can be interpreted as the respective optimal coupon and CDS protection when  $\delta_0 = 1$ . Note that this also implies that  $\theta(c_B) = \frac{\eta}{\gamma} c_B$  is linear in  $c_B$ .

To show this, simply notice that given  $c_B = \gamma \delta_0$  and  $T = \rho \delta_0$ , the optimal CDS protection is linear in  $\delta_0$ :

$$\begin{split} \theta &= \arg \max_{\tilde{\theta}} b(\delta_0; c_B, \theta, T) - C(\delta_0; c_B, \theta, T) \\ &= \arg \max_{\tilde{\theta}} \delta_0 b(1; c_B/\delta_0, \tilde{\theta}/\delta_0, T/\delta_0) - \delta_0 C(1; c_B/\delta_0, \tilde{\theta}/\delta_0, T/\delta_0) \\ &= \delta_0 \arg \max_{\tilde{\eta}} b(1; \gamma, \tilde{\eta}, \rho) - C(1; \gamma, \tilde{\eta}, \rho) \\ &= \delta_0 \eta. \end{split}$$

Similarly, given  $\theta = \eta \delta_0$  and  $T = \rho \delta_0$ , the optimal coupon is linear in  $\delta_0$ :

$$\begin{split} c_B &= \arg \max_{\tilde{c}_B} b(\delta_0; c_B, \theta, T) - C(\delta_0; c_B, \theta, T) + E(\delta_0; c_B, \theta, T) \\ &= \arg \max_{\tilde{c}_B} \delta_0 b(1; \tilde{c}_B / \delta_0, \theta / \delta_0, T / \delta_0) - \delta_0 C(1; \tilde{c}_B / \delta_0, \theta / \delta_0, T / \delta_0) + \delta_0 E(1; \tilde{c}_B / \delta_0, \theta / \delta_0, T / \delta_0) \\ &= \delta_0 \arg \max_{\tilde{\gamma}} b(1; \tilde{\gamma}, \eta, \rho) - C(1; \tilde{\gamma}, \eta, \rho) + E(1; \tilde{\gamma}, \eta, \rho) \\ &= \delta_0 \gamma. \end{split}$$

**Step 3.** We claim that the collection of thresholds is linear in  $\delta_0$ . That is,

$$T = (\delta_n, \delta_i, \delta_d) = (\nu \delta_0, \iota \delta_0, \lambda \delta_0) = (\nu, \iota, \lambda) \, \delta_0 = \rho \delta_0$$

where  $\nu, \iota$ , and  $\lambda$  are constants independent upon  $\delta_0$ , which can be interpreted as the respective optimal renegotiation, investment, and bankruptcy threshold when  $\delta_0 = 1$ . This also implies that these thresholds are linear in  $\theta$  and  $c_B$ .

To prove the claim, we substitute  $\iota \delta_0$  for  $\delta_i$  and  $\lambda \delta_0$  for  $\delta_d$  in the expressions of  $\frac{\partial P_d^0(\delta_d)}{\partial \delta} \delta_d$ ,  $\frac{\partial P_d^i(\delta_i)}{\partial \delta} \delta_i$ ,  $\Gamma'(\delta_d, \delta_d)$ , and  $\Gamma_i(\delta_i, \delta_d)$ . After cancelling out  $\delta_0$ , we find that these functions do not depend upon  $\delta_0$ . These results, together with  $\theta = \lambda \delta_0$ ,  $\delta_i = \iota \delta_0$ , and  $\delta_d = \lambda \delta_0$ , yield that both sides of equations (18) and those of (19) are linear in  $\delta_0$ .

Dividing both sides of the equations by  $\delta_0$ , we obtain a non-linear system of equations of  $(\iota, \lambda)$  that is independent of  $\delta_0$ . This verifies the existence of a pair of  $(\delta_i = \iota \delta_0, \delta_d = \lambda \delta_0)$  solving the original non-linear system of equations (18) and (19). Moreover, given  $c_B = \gamma \delta_0$  and  $\theta = \lambda \delta_0$  from step 2, it is evident that  $\delta_n$ , as given by equation (11), is linear in  $\delta_0$ . That is,  $\delta_n = \nu \delta_0$ .

Combing all three steps, we can see that the value functions of equity, debt, and the total firm are homogenous of degree one in  $\delta_0$  under the optimal choices of  $c_B$ ,  $\theta$ , and T.<sup>38</sup>

**Proof of Proposition 5.** To simplify the equations, we assume that  $\tau = 0$  so that  $\tilde{\phi} = \phi$ . When the firm always invests, the value of the CDS contract to the debt holders is  $CDS(\delta) = \theta \left(\frac{\delta}{\delta_d}\right)^{z_1} - \frac{p}{r} \left(1 - \left(\frac{\delta}{\delta_d}\right)^{z_1}\right)$ . Let  $\tilde{B}(\delta)$  be the value of the debt contract without CDS protection. For  $\delta \in [\delta_d, \delta_n]$ , the debt's portfolio value is

$$D(\delta) = \underbrace{\frac{1 - \phi i}{r - (\mu + i)} \delta + \left(L\delta_d - \frac{1 - \phi i}{r - (\mu + i)}\delta_d\right) \left(\frac{\delta}{\delta_d}\right)^{z_1}}_{\equiv \tilde{B}(\delta)} + CDS(\delta).$$

Since the bankruptcy time is chosen by the debt holders effectively, the smooth-pasting condition,  $D'(\delta_d) = L$ , characterizes the bankruptcy threshold:  $\delta_d = \frac{z_1}{z_1-1} \left(\theta + \frac{p}{r}\right) / \left(\frac{1-\phi i}{r-(\mu+i)} - L\right)$ . For  $\delta > \delta_n$ , the debt's portfolio value is given by

$$D(\delta) = \frac{c_B}{r} + \left(\tilde{b}(\delta_n) - \frac{c_B}{r}\right) \left(\frac{\delta}{\delta_n}\right)^{z_1} + CDS(\delta).$$

Moreover, the equity value is zero on  $[\delta_d, \delta_n]$  and  $E(\delta) = \frac{1-\phi i}{r-(\mu+i)}\delta - \frac{c_B}{r} + \left(\frac{c_B}{r} - \frac{1-\phi i}{r-(\mu+i)}\delta_n\right)\left(\frac{\delta}{\delta_n}\right)^{z_1}$  for  $\delta > \delta_n$ . Smooth-pasting of the equity value,  $\lim_{\delta \uparrow \delta_n} E'(\delta) = \lim_{\delta \downarrow \delta_n} E'(\delta)$ , implies that  $\delta_n = \frac{z_1}{z_1-1}\frac{c_B}{r}\frac{r-(\mu+i)}{1-\phi i}$ . We now argue that in maximizing the ex-ante debt's portfolio value  $D(\delta_0)$ , it is optimal for the debt holders to choose  $\theta = 0$ . First, the independence of  $\delta_n$  with respect to  $\theta$  and the fact that  $CDS(\delta_0) = 0$  implies that  $D(\delta_0)$  is independent of  $\theta$  for  $\delta_0 > \delta_n$ . Hence the debt holders have weak incentives to choose  $\theta = 0$ .

<sup>&</sup>lt;sup>38</sup>We show there exists a linear equilibrium exhibiting the scale invariance property. However, as is well known in the literature, we are unable to rule out any other non-linear equilibria.

Second, for  $\delta_0 \in [\delta_d, \delta_n]$ ,  $D(\delta_0) = \tilde{B}(\delta_0)$  and it is decreasing in  $\delta_d$  because under our parametric restrictions,  $\frac{1-\phi i}{r-(\mu+i)} > L$ . By the competitive pricing condition (*p* is increasing in  $\theta$ ), the bankruptcy threshold  $\delta_d$  is increasing in  $\theta$ . So, a higher  $\theta$  reduces the debt value. Therefore, it is optimal for the debt holders not to hedge. It follows that  $\delta_d = 0$  and the fact that  $\theta = 0$ , together with proposition 2, implies there is no debt overhang.

Appendix for Section 5.2. With the specification of the renegotiation friction, the choices of the equity holders remain to be the thresholds  $(\delta_d, \delta_n, \delta_i)$ . Given the thresholds, the debt valuation remains the same as in Section 3.2.1 because the debt does not bear the renegotiation cost. Given the baseline equity value  $E(\delta)$ , the general solution of the equity value with renegotiation frictions  $E^q(\delta)$  is given by

$$E^{q}(\delta) = \begin{cases} (1-\tau) \left(\frac{1-\tilde{\phi}i}{r-(\mu+i)}\delta - \frac{c_{B}}{r}\right) + A^{q}\delta^{z_{1}} & \text{if } \delta > \delta_{n};\\ (1-q) \cdot E(\delta) & \text{if } \delta_{d} \le \delta \le \delta_{n}. \end{cases}$$

The coefficient  $A^q$  is pinned down by the value-matching condition at  $\delta_n$ :  $E^q(\delta_n) = (1 - q) \cdot E(\delta)$ . With some algebra, we can show that for  $\delta > \delta_n$ ,

$$E^{q}(\delta) = (1-\tau)\left(U_{i}\delta - \frac{c_{B}}{r}\right) + (1-\tau)\left(\frac{c_{B}}{r} - (1-q)\theta - (L+q(U_{i}-L))\delta_{n}\right)\left(\frac{\delta}{\delta_{n}}\right)^{z_{1}} + (1-q)O(\delta_{n})\left(\frac{\delta}{\delta_{n}}\right)^{z_{1}},$$

where  $O(\delta) \equiv (1-\tau) \left(\theta + L\delta_d - \frac{\delta_d}{r-\mu}\right) P_d^i(\delta) + (1-\tau) \Pi \delta_i \cdot \Gamma_i(\delta_i, \delta_d) \left(\frac{\delta}{\delta_i}\right)^{z_1}$  is the option value in (9). Notice that the smooth-pasting condition,  $E^{q'}(\delta_d) = 0$ , is equivalent to  $E'(\delta_d) = 0$ ; and the maximization of (15) with respect to  $\delta_i$  requires  $E^{q'}(\delta_i) = (1-q)\phi$ , which is equivalent to  $E'(\delta_i) = \phi$ . Therefore, equations (21) and (20) still determine both  $\delta_i$  and  $\delta_d$  simultaneously. The smooth-pasting condition at  $\delta_n$  characterizes the optimal renegotiation boundary and gives (16).

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