Crop Insurance under Restricted Access to Financial Markets

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Abstract

Optimal farmer's behavior in the presence of subsidized crop insurance and incomplete financial markets is analyzed. Because of bounded rationality, market frictions, and various market failures, farmers and insurers have access to different closed subsets of the financial market span. Therefore, farmers and insurers evaluate the crop insurance against alternatives in their respective subsets of the market span. Because farmers can price only the orthogonal projection of the crop insurance on their market span, their willingness to pay for the insurance is lower than the insurers' willingness to accept. The crop insurance subsidy helps bridge the gap between farmers and insurers valuations. In this setup, the subsidized crop insurance influences farmers' production choices by altering both the relative returns across states of Nature and their marginal risk attitudes. However, these changes should not be mistakenly attributed to changes in preferences over risk, but to changes in the available financial opportunities.

The subsidized crop insurance program has a long history in the United States. The discourse surrounding the implications of governmental support for crop insurance points in two directions. Firstly, supporters see the crop insurance as a critical risk management safety net for producers (Risk Management Agency, 2017). Meanwhile, concerns are voiced that subsidizing crop insurance creates incentives for farmers to change their risk behaviour and alter production decisions (Nixon, 2012; Babcock, 2016).

This challenge is not new and various arguments were proposed to explain or to reconcile these beliefs. Among them, it was recognized that crop-insurance is subject to adverse selection and moral hazard (Skees and Reed, 1986; Nelson and Loehman, 1987; Chambers, 1989; Miranda, 1991; Goodwin, 1993; Just, Calvin, and Quiggin, 1999; Makki and Somwaru, 2001) and systemic risk (Miranda and Glauber, 1997; Barnett, 2000). Alternatively, governmental supported crop insurance operates as an income transfer to the farmers (Just, Calvin, and Quiggin, 1999; Goodwin and Smith, 2013).

A distinct approach is embodied by the work of Chambers (2007), Chambers and Quiggin (2009) and Chambers (2018). They make an essential observation. In addition to crop insurance, farmers in developed countries have access to other risk mitigating tools (i.e. financial markets, off-farm work, storage). Thus, the crop insurance is evaluated against other risk management alternatives for a share of their budget. In this setup, due to the separation between production and consumption decisions, the subsidized crop insurance affects farmers' production choices by altering relative returns across states of Nature, but not their marginal risk attitudes.

While spot on, their conclusions assume that farmers and potential insurers have access to the same crop insurance alternatives (i.e. financial market span). We relax this assumption by allowing farmers and insurers to access closed subsets of the entire market span. Therefore, farmers and insurers will evaluate the crop insurance against other alternatives in their respective subsets of the market span. Because farmers can price only the orthogonal projection of the crop insurance on their market span, their willingness to pay for the insurance is lower than the insurers' willingness to accept. The crop insurance subsidy helps bridge the gap between farmers and insurers valuations. In this setup, the subsidized crop insurance influences farmers' production choices by altering both the relative returns across states of Nature and their marginal risk attitudes.

We proceed by providing a theoretical model of a rational decision maker, mnemonically called the farmer, who maximizes consumption over two periods in the presence of a stochastic agricultural technology and incomplete financial markets. Preferences over stochastic consumption are assumed to be strictly monotonic, but no other functional specification is imposed. In addition, the farmer can purchase a crop insurance that is subsidized at a decreasing rate. Everything else constant, for the purpose of consumption, farmers are indifferent between income generated from the agricultural production or off-farm activities, such as financial markets. Farmers, as rational agents, act to eliminate any arbitrages between farm and off-farm activities while maximizing their utility over stochastic consumption.

We show that crop insurance choices are motivated by profit maximizing behavior and not risk mitigation considerations. However, because farmers and insurers have access to different subsets of the market span, their marginal risk valuations differ. The subsidized crop insurance will enrich the farmers' market span altering the marginal risk valuation, in addition to changing the relative returns across states of Nature.

The paper is structured as follows. The model set up and the producer optimal behavior under restricted access to financial markets are flesh out in the first two sections. The third section details the valuation of a stochastic consumption vector priced outside the farmer's market span, while the forth section discusses the effects of expanding the farmer's market span. With these in hand, a fifth section analyzes the effects of subsidized crop insurance on farmer's production and consumption decisions. A final section concludes the paper.

1 Theoretical Model

The setup is similar to Chambers and Voica (2017) and Chambers (2018). A rational agent, whom we mnemonically refer to as the farmer, maximizes consumption over two periods. The first period (the decision period), 0, involves no uncertainty. The second period, 1, is uncertain. Uncertainty is modeled by a finite state space, described by a finite set, Ω, where each element of Ω, referred to as a state, is a complete and mutually exclusive description of the world.[1] For example, in a two-states representation of the world, a state could be "rain" and another could be "no rain". Uncertainty is resolved by Nature, choosing from Ω. That choice, however, is only revealed to the farmer after the farmer's choices have been made in period 0. Without loss of generality, $\Omega = \{1, \ldots, S\}$.

The farmer is competitive and takes inputs and state-contingent output prices as given. Preferences over consumption in the two periods, $q_0 \in \mathbb{R}_+$ and $q_1 \in \mathbb{R}^{\Omega}_+$, are continuous and strictly increasing in each argument, and represented by $W(q_0, q_1)$. The initial wealth endowment, $\omega > 0$, is nonstochastic. In period 0, the farmer can undertake agricultural production activities and trade financial assets that generate state-contingent income in period 1. Production is characterized by a stochastic technology. In period 0, the farmer chooses the level of state-contingent period 1 output $z \in \mathbb{R}^{\Omega}_+$, where the *ex post* realization of the output in state s is z_s. The associated variable cost is $c(w, z)$ where $w \in \mathbb{R}^N_{++}$ is the vector of variable input prices in period 0.[2] Cost is assumed to be convex in z.

The farmer can also buy and sell assets in financial markets. These markets allow rational agents to trade financial assets at a period 0 price $v \in \mathbb{R}^J_{++}$ and promise an

[1]The theoretical framework used here is the state-contingent approach to uncertainty. An accessible treatment to the state-contingent approach is Chambers and Quiggin (2000).

[2]For an axiomatic study of cost functions see Chambers and Quiggin (2000).

ex ante stochastic period 1 payoff given by the $\Omega \times J$ matrix A. For example, the jth financial asset trades at period 0 price $v_j \in \mathbb{R}_{++}$ and delivers the stochastic period 1 payoff $A_j \in \mathbb{R}^\Omega$ (the jth column of the payoff matrix A), where the period 1 payoff in state s is $A_{js} \in \mathbb{R}$. The number of units of this asset purchased in period 0 is denoted $h_j \in \mathbb{R}$, while the entire financial market portfolio is denoted by $h \in \mathbb{R}^J$. Without loss of generality, the payoff matrix A is assumed to have full column rank J (i.e. the column vectors are linearly independent) and $J < |\Omega|$ (i.e. financial markets are incomplete).

The financial opportunities are represented by the linear subspace, $\mathcal{M} \subset \mathbb{R}^\Omega$, spanned by the column vectors of the matrix A, where $\mathcal{M}$ is defined by

$$\mathcal{M} = \{y \in \mathbb{R}^\Omega : Ah = y, h \in \mathbb{R}^J\}$$

Any stochastic consumption vector q_1 that falls in $\mathcal{M}$ can be replicated and priced in the financial market. However, bounded rationality, information asymmetries, market frictions and various market failures may prevent the farmer from accessing all the financial assets available in the market. Thus, the farmer may be able to access only a smaller subspace, $\mathcal{M}_\mathcal{F} \subset \mathbb{R}^\Omega$, of the financial market $\mathcal{M}$ (i.e. $\mathcal{M}_\mathcal{F} \subseteq \mathcal{M}$).

Similarly, the farmer's financial opportunities subspace $\mathcal{M}_\mathcal{F}$ is defined as

$$\mathcal{M}_\mathcal{F} = \{y \in \mathbb{R}^\Omega : A_F h_F = y, h_F \in \mathbb{R}^F\}$$

where $F \leq J$, A_F is a $\Omega \times F$ matrix of *ex ante* financial payoffs in period 1, h_F is the farmer's portfolio holdings vector and v_F is the period 0 price of financial assets available to the farmer. Any stochastic consumption vector that falls in $\mathcal{M}_\mathcal{F}$ can be replicated and priced by the F financial assets, but since $\mathcal{M}_\mathcal{F} \subseteq \mathcal{M}$, it can also be replicated and priced in $\mathcal{M}$. The reverse is not always true. The farmer may not be able to replicated stochastic consumption vectors that fall in $\mathcal{M} \setminus \mathcal{M}_\mathcal{F}$. These risks fall in the orthogonal complement of $\mathcal{M}_{\mathcal{F}}$, ${\mathcal{M}_{\mathcal{F}}}^{\perp}$.

1.1 Optimal Behavior and Equilibrium

The farmer's period 0 problem is to choose $q_0 \in \mathbb{R}_+$, $q_1 \in \mathbb{R}^{\Omega}_+$, $z \in \mathbb{R}^{\Omega}_+$ and $h_F \in \mathbb{R}^F$ to[3]

$$\max \Big\{ W\Big(q_0, q_1\Big) : q_0 \le \omega - c(w,z) - v_F^T h_F, \qquad (1)$$
$$q_1 \le pz + A_F h_F \Big\}$$

In words, the farmer maximizes consumption over the two periods subject to $|\Omega|+1$ budget constraints and the restriction to operate in $\mathcal{M}_{\mathcal{F}}$. In the first period, the consumption can not exceed the difference between the initial wealth ω and the cost of assembling the second period consumption via the agricultural production, $c(w,z)$, and financial market participation, $v_F h_F$. In the second period, the consumption is bounded by the agricultural revenue, pz, and the financial payoff, $A_F h_F$.

The constraints are binding because preferences are strictly increasing in consumption. Hence, for optimal values of second period consumption and output levels, q_1 and z, the optimal level of financial market participation is

$$h_F = P_F(q_1 - pz) \qquad (2)$$

where $P_F = (A_F^T A_F)^{-1} A_F^T$. After substituting for h_F in (1) yields:

$$\max_{q_1} \Big\{ W\Big(\omega - v_F^T P_F q_1 - \Pi(p, w, v_F^T P_F), q_1\Big)\Big\} \qquad (3)$$

and

$$\Pi(p, w, v_F^T P_F) = \max_z \Big\{ v_F^T P_F pz - c(w,z) \Big\}$$

[3] A richer decision environment is possible. However, it will complicate the notation without changing the predictions of the model.

The solution to (3) extends the separation results in Chambers and Quiggin (2009), Chambers and Voica (2017) and Chambers (2018) to the present context.[4] Independent of their risk preferences, farmers agree at the margin on the value of consumption by eliminating any arbitrages between physical production and financial markets. However, they do not use the stochastic discount factor corresponding to the entire financial market, $v^T P = v^T (A^T A)^{-1} A^T$, but the stochastic discount factor induced by the assets in $\mathcal{M}_{\mathcal{F}}$, $v_F^T P_F = v_F^T (A_F^T A_F)^{-1} A_F^T$. This is simply because the farmers can access $\mathcal{M}_{\mathcal{F}}$ and not $\mathcal{M}$.

Assuming the preference and the cost function are differentiable, the first order conditions for problem (3), at interior solutions, are:

$$\frac{\partial W(q_0, q_1)/\partial q_{1s}}{\partial W(q_0, q_1)/\partial q_0} = v_F^T P_{Fs}, \ \forall s \in \Omega \tag{4}$$

$$\frac{\partial c(w, z)}{\partial z_s} = v_F^T P_{Fs} p_s, \ \forall s \in \Omega \tag{5}$$

The first order conditions (4) and (5) are pricing equations. In equation (4), the marginal rate of substitution between consumption in state s and consumption in period 0 equals the stochastic discount factor for the state s, $v_F^T P_{Fs}$. From (5), the marginal cost of increasing consumption by one unit in the state s equals the price of the output in that state, p_s, times the stochastic discount factor for the state s, $v_F^T P_{Fs}$. Hence, their production and consumption decisions are separated. Farmers, with different preferences over risk, but with access to the same market span, will make, at the margin, the same consumption and production decisions.

However, in both (4) and (5), farmers use $v_F^T P_{Fs}$ and not $v^T P_s$ to guide their consumption and production decisions. As long as $v_F^T P_{Fs} \neq v^T P_s$, their consumption and production decisions will depend on their financial opportunities, $\mathcal{M}_{\mathcal{F}}$. Two

[4] The proof of the separation result is similar to the one in Chambers and Quiggin (2009), Chambers and Voica (2017) and Chambers (2018). Thus, for brevity, the proof is omitted. The interested reader is encouraged to consult the papers mentioned above.

rational agents, with the same preferences over risk, will disagree at the margin on their consumption and production decision if they have access to financial markets with different stochastic discount factors. These differences may be mistakenly attributed to preferences over risk, when, in fact, they are due to different financial market opportunities.

1.2 Valuing the Stochastic Consumption

A natural next question is how does the valuation of a stochastic consumption q depend on the financial market opportunities available? To answer this question consider first a consumption vector q outside the farmer's market span $\mathcal{M}_{\mathcal{F}}$, but within the financial market $\mathcal{M}$, $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$. From the farmers' point of view, the stochastic consumption vector q can not be replicated in their market span because it falls outside $\mathcal{M}_{\mathcal{F}}$. However, from the market's point of view, q is completely replicable and priced because it falls in $\mathcal{M}$. For any $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}} \subset \mathcal{M}$, there is an unique portfolio $h \in \mathbb{R}^J$ such that

$$q = Ah \tag{6}$$

To obtain h pre-multiply (6) by $(A^T A)^{-1} A^T$ to get

$$h = (A^T A)^{-1} A^T q = Pq \tag{7}$$

The period 0 cost of assembling the period 1 consumption vector $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$ is $v^T h$ and the period 0 value of q is $v^T Pq$. As a consequence of *the Law of One Price*, $v^T h = v^T Pq$ (Cochrane, 2001; LeRoy and Werner, 2001). Thus, the market values the period 1 stochastic consumption $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$ at $v^T Pq$ in period 0 prices. The question is how much is the farmer willing to pay for it?

The consumption vector q falls outside the farmer's market span $\mathcal{M}_{\mathcal{F}}$, which is a closed subspace of $\mathcal{M}$. By the Hilbert Projection Theorem, there exits a unique $q_F \in \mathcal{M}_{\mathcal{F}}$ such that $q = q_F + (q - q_F)$ and $q - q_F \in \mathcal{M}_{\mathcal{F}}{}^{\perp}$.[5] In words, q_F is the orthogonal projection of q onto $\mathcal{M}_{\mathcal{F}}$ and it is determined as the *Least Square Estimate* of q

$$q_F = A_F P_F q \tag{8}$$

from where it follows that

$$q - q_F = q - A_F P_F q = (I_\Omega - A_F P_F)q \tag{9}$$

where I_Ω is the $|\Omega|$ dimensional identity matrix and $P_F = (A_F^T A_F)^{-1} A_F^T$. Observe that $P_F(q - q_F) = P_F(I_\Omega - A_F P_F)q = 0$ because $P_F A_F = I_\Omega$. Thus, the farmer's period 0 valuation of q in $\mathcal{M}_{\mathcal{F}}$ is

$$v_F^T P_F q = v_F^T P_F q_F + v_F^T P_F (q - q_F) = v_F^T P_F q_F \tag{10}$$

The stochastic discount factor used by the farmer to price q in $\mathcal{M}_{\mathcal{F}}$, $v_F^T P_F$, is different than the stochastic discount factor used to price q in $\mathcal{M}$, $v^T P$. Also, only the orthogonal projection of q on $\mathcal{M}_{\mathcal{F}}$, q_F, is actually priced. Recycling previous arguments, the period 0 value of the q in $\mathcal{M}$ is

$$v^T P q = v^T P(q_F + (q - q_F)) = v_F^T P_F q_F + v^T P(q - q_F) > v_F^T P_F q_F \tag{11}$$

which says that the market valuation of q in $\mathcal{M}$ is higher than what the farmer is willing to pay for it in $\mathcal{M}_{\mathcal{F}}$.[6] Thus, without government intervention, no trade is possible because the farmer's willingness to pay for q is lower than the market willingness to accept. However, for a subsidy equal to the difference between the

[5] The interested reader is referred to Luenberger (1969) for the formal statement of the Classical Projection Theorem and its subsequent application.

[6] The financial subspace $\mathcal{M}$ can be written as a direct sum of $\mathcal{M}_{\mathcal{F}}$ and its orthogonal complement $\mathcal{M}_{\mathcal{F}}^{\perp}$, $\mathcal{M} = \mathcal{M}_{\mathcal{F}} \oplus \mathcal{M}_{\mathcal{F}}^{\perp}$. For strictly positive period 0 prices, it follows $v^T P(q - q_F) > 0$.

market price $v^T Pq$ and the farmers valuation $v_F^T P_F q = v_F^T P_F q_F$, the farmer would be willing to purchase the asset $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$.

Equally important is the case of a stochastic consumption q falling outside the market span $\mathcal{M}$. Now, a potential seller of q, with access to $\mathcal{M}$, can not replicate the consumption vector q in $\mathcal{M}$. Instead, q will be valued in $\mathcal{M}$ at $v^T h^\star$ where

$$h^\star = \underset{h}{\operatorname{argmin}} \left\{ v^T h : q \leq Ah \right\} \tag{12}$$

Meanwhile, the farmer's valuation of the consumption vector q in $\mathcal{M}_{\mathcal{F}}$ still equals $v_F^T P_F q_F$. However, the difference between the willingness to accept of a rational agent in $\mathcal{M}$ and the willingness to pay of the farmer has increased. Observe that a $q \notin \mathcal{M}$ can be decomposed as

$$q = q_F + (q - q_F) = q_F + (q_{\mathcal{M}} - q_F) + (q - q_{\mathcal{M}}) \tag{13}$$

where q_F and $q_{\mathcal{M}}$ are the orthogonal projections of the stochastic consumption vector q on farmer's span $\mathcal{M}_{\mathcal{F}}$ and on the market span $\mathcal{M}$, respectively. Hence,

$$\begin{aligned} v^T h^\star \geq v^T Pq = v^T P(q_{\mathcal{M}} + (q - q_{\mathcal{M}})) = v^T P q_{\mathcal{M}} \\ = v^T P(q_F + (q - q_F)) = v_F^T P_F q_F + v^T P(q - q_F) > v_F^T P_F q_F \end{aligned} \tag{14}$$

For a stochastic consumption vector $q \notin \mathcal{M}$, the subsidy needed to bridge the gap between the farmer's willingness to pay and the seller's willingness to accept is higher than for an asset in the market span $\mathcal{M}$.

1.3 Expanding the Farmer's Financial Opportunity Set

Granting access to the financial asset $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$ does not mean the farmers gain access to the entire market span $\mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$, but only to the market span generated by q, $\mathcal{M}_q$, where

$$\mathcal{M}_q = \{y \in \mathbb{R}^{\Omega} : y = \alpha q, \alpha \in \mathbb{R}_+\} \subseteq \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$$

The new market span available to farmers is

$$\mathcal{M}_{F+q} = \{y \in \mathbb{R}^{\Omega} : y = A_F h_F + \alpha q, h_F \in \mathbb{R}^F \text{ and } \alpha \in \mathbb{R}_+\}$$

where $\mathcal{M}_F \subset \mathcal{M}_{F+q}$ reflecting an expansion in the farmers' financial opportunities set. For a subsidy that covers the difference between the market price and the farmer's willingness to pay, the farmer solves the problem

$$\max \Big\{ W\Big(q_0, q_1\Big) : q_0 \leq \omega - c(w, z) - v_F^T h_F - \alpha v_F^T P_F q, \quad q_1 \leq pz + A_F h_F + \alpha q \Big\} \tag{15}$$

where α is the amount of the asset q purchased. Problem (15) is equivalent to offering the farmers a subsidized pure-income insurance product. With a slight change of notation, following previous arguments, the optimal financial portfolio is

$$h_{F+q} = P_{F+q}(q_1 - pz)$$

where $h_{F+q} = [h_F, \alpha]$, $P_{F+q} = (A_{F+q}^T A_{F+q})^{-1} A_{F+q}^T$ and $A_{F+q} = [A_F, q]$. After substituting for h_{F+q} in the first period consumption of (15), the farmer solves

$$\max_{q_1} \Big\{ W\Big(\omega - \tilde{v}_{F+q}^T P_{F+q} q_1 + \Pi(p, w, \tilde{v}_{F+q}^T P_{F+q}), q_1\Big)\Big\} \tag{16}$$

and

$$\Pi(p, w, \tilde{v}_{F+q}^T P_{F+q}) = \max_z \Big\{ \tilde{v}_{F+q}^T P_{F+q} pz - c(w, z)\Big\}$$

where $\tilde{v}_{F+q} = [v_F, v_F^T P_F q] \neq [v_F, v^T Pq] = v_{F+q}$. Farmers obtain access to an additional asset $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$, for which they pay the value of its projection on $\mathcal{M}_{\mathcal{F}}$, $v_F^T P_F q$, and not the market value $v^T Pq$. The additional financial asset changes the farmer's stochastic discount factor, thus the optimal production. Separation between production and consumption decisions still holds, but the optimal agricultural output depends on the new stochastic discount factor. This can be observed from the first order conditions:

$$\frac{\partial W(q_0, q_1)/\partial q_{1s}}{\partial W(q_0, q_1)/\partial q_0} = \tilde{v}_{F+q}^T P_{F+q,s}, \ \forall s \in \Omega \tag{17}$$

$$\frac{\partial c(w, z)}{\partial z_s} = \tilde{v}_{F+q}^T P_{F+q,s} p_s, \ \forall s \in \Omega \tag{18}$$

As in the case of the first order conditions (4) and (5), farmer's consumption and production choices are separated, at the margin, by the financial market via the stochastic discount factor $\tilde{v}_{F+q}^T P_{F+q}$. However, while production decisions are indeed independent of the farmer's risk preferences, they do depend on the new stochastic discount factor $\tilde{v}_{F+q}^T P_{F+q}$. Thus, subsidizing a pure-income insurance or any other similar financial instruments changes the farmer's marginal output choices. These changes do not reflect a switch in the farmer's risk preferences, which remain separated from the output choices, but a change in the financial opportunities available to her. Hence, the changes in farmer's output choices should not be mistakenly attributed to a change in the farmer's risk preferences due to the availability of a pure-income insurance, but to the expanse of the financial opportunities. For farmers, the pure-income insurance is not a risk mitigating tool, but a risk free opportunity to increase their profit. As any other rational agents, farmers would not pass the chance to increase their profits. The expansion of their financial span provides exactly this opportunity.

Next, we consider the case of $q \notin \mathcal{M}$. While the implications are slightly different, because the mathematical details are the same as in the case of $q \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$, the exposition is kept short. For an asset $q \notin \mathcal{M}$, $\mathcal{M}_q \nsubseteq \mathcal{M}$, hence the new market span available to farmers $\mathcal{M}_{F+q} \nsubseteq \mathcal{M}$. Farmers receive access to a stochastic asset outside the market span $\mathcal{M}$. The asset is priced at $v^T h^\star$ in $\mathcal{M}$, but is purchased by farmers at the value of its projection on the market span $\mathcal{M}_{\mathcal{F}}$, $v_F^T P_F q_F$.

For a subsidy that covers the difference between $v^T h^\star$ and $v_F^T P_F q_F$, the farmer solves (15) and observes the separation result in (16). However, because $q = q_{\mathcal{M}} + q_{\mathcal{M}^\perp}$, the new *ex ante* stochastic period 1 payoff matrix is given by $A_{F+q} = [A_F, q_{\mathcal{M}} + q_{\mathcal{M}^\perp}]$, as compared to $A_{F+q} = [A_F, q_{\mathcal{M}}]$ when $q \in \mathcal{M}$, while the portfolio stays $h_{F+q} = [h_F, \alpha]$. For any positive (negative) value of α, the farmer acquires(sells) αq, which is equivalent to a fix proportion bundle of $(\alpha q_{\mathcal{M}}, \alpha q_{\mathcal{M}^\perp})$.

1.4 Subsidized Crop Insurance

Consider a crop insurance $\gamma : \mathbb{R}_+^{\Omega+1} \to \mathbb{R}_+^{\Omega}$ that pays the stochastic indemnity $\gamma(I, pz)$ in period 1 for a period 0 price of $v(I, pz)$, where $I \in R_{++}$ is the insurance trigger threshold and the indemnity $\gamma(I, pz)$ has the following functional form.[7]

$$\gamma(I, pz) = (\max\{I - p_1 z_1, 0\}, \ldots, \max\{I - p_S z_S, 0\}) \tag{19}$$

In what follows, we assume that the crop insurance $\gamma(I, pz)$ can not be replicated in the financial market span $\mathcal{M}$ (i.e. $\gamma(I, pz) \notin \mathcal{M}$), and it is sold by insurers with access to a subset $\mathcal{M}_{\mathcal{I}}$ of the market span $\mathcal{M}$ (i.e. $\mathcal{M}_{\mathcal{I}} \subseteq \mathcal{M}$) such that $\mathcal{M}_{\mathcal{I}} \setminus \mathcal{M}_{\mathcal{F}} \neq \emptyset$.[8]

[7] Alternative specifications of the stochastic indemnity are readily available. For example, instead of individual state contingent revenue, an average region state contingent revenue can be used as trigger for the insurance payments. Furthermore, the revenue used as a trigger for insurance payment can be assumed not to be under the choice of the producers. While these alternative functional forms of the stochastic indemnity are interesting in their own right, they are only secondary to the point we are trying to make here. Hence, for clarity of exposition, we contain our analysis to the stochastic indemnity described in (19).

[8] The case $\mathcal{M}_{\mathcal{I}} = \mathcal{M}_{\mathcal{F}}$ was discussed by Chambers (2018), while $\gamma(I, pz) \in \mathcal{M}_{\mathcal{I}} \cap \mathcal{M}_{\mathcal{F}}$ is equivalent to $\mathcal{M}_{\mathcal{I}} = \mathcal{M}_{\mathcal{F}}$. We are interested in the case where the insurance is priced outside farmers' financial market span. The current setup includes as special cases $\gamma(I, pz) \in \mathcal{M} \setminus \mathcal{M}_{\mathcal{F}}$, hence $\gamma(I, pz) \in \mathcal{M}_{\mathcal{I}} \setminus \mathcal{M}_{\mathcal{F}}$ and

Insurers must fulfill their contractual obligations to the farmers when the insurance coverage is triggered, hence the indemnity schedule must be at least covered, if not replicated, in their market span $\mathcal{M}_{\mathcal{I}}$. As such, a potential insurer with access to $\mathcal{M}_{\mathcal{I}}$ will price the insurance at $v_{\mathcal{I}}^T h_{\mathcal{I}}^\star$, where $h_{\mathcal{I}}^\star = \text{argmin}\{v_{\mathcal{I}}^T h_{\mathcal{I}} : \gamma(I, pz) \leq A_{\mathcal{I}} h_{\mathcal{I}}\}$. By the *law of one price*, $v(I, pz) = v_{\mathcal{I}}^T h_{\mathcal{I}}^\star$, and the insurance will be traded if $v_F^T P_F \gamma(I, pz) + a(I, pz) \geq v_{\mathcal{I}}^T h_{\mathcal{I}}^\star$, where $v_F^T P_F \gamma(I, pz)$ is the farmer's willigness to pay for the insurance and $a(I, pz)$ is the government subsidy.

Without loss of generality, the values of agricultural output in the first k states of Nature are assumed to be lower than the indemnity, $I > p_s z_s$ for $s \leq k$, and higher or equal in the remaining states, $I \leq p_s z_s$ for $s > k$. Hence the insurance becomes

$$\begin{aligned}\gamma(I, pz) =& (I - p_1 z_1, \ldots, I - p_k z_k, 0_{k+1}, \ldots, 0_S)) \\ =& (q_{\mathcal{I}} + q_{\mathcal{I}}^\perp - p_1 z_1, \ldots, q_{\mathcal{I}} + q_{\mathcal{I}}^\perp - p_k z_k, 0_{k+1}, \ldots, 0_S))\end{aligned}$$

where $q_{\mathcal{I}}$ is the orthogonal projection of I on $\mathcal{M}_{\mathcal{I}}$ and $q_{\mathcal{I}}^\perp$ is the orthogonal complement.

As before, the farmer will be able to price only the orthogonal projection of insurance $\gamma(I, pz)$ over the market span $\mathcal{M}_{\mathcal{F}}$. In the presence of a subsidy $a(I, pz)$ that covers at least the difference between the market price of $\gamma(I, pz)$ in $\mathcal{M}_{\mathcal{I}}$ and the market price of $\gamma(I, pz)$ in $\mathcal{M}_{\mathcal{F}}$, the farmer's problem is to choose $q_0 \in \mathbb{R}_+$, $q_1 \in \mathbb{R}_+^\Omega$, $z \in \mathbb{R}_+^\Omega$, $h_F \in \mathbb{R}^F$ and $I \in \mathbb{R}$ to

$$\begin{aligned}\max \Big\{ W\Big(q_0, q_1\Big) : q_0 \leq \omega - c(w, z) - v_F^T h_F - \alpha v_F^T P_F \tilde{I} + v_F^T P_F \tilde{pz} \\ q_1 \leq pz + A_F h_F + \alpha \tilde{I} - \tilde{pz} \Big\}\end{aligned} \tag{20}$$

where α is a scalar, $\tilde{I}$ is the vector with values I/α for the first k states of Nature, and 0 otherwise, $\tilde{pz}$ is the vector with values $p_s z_s$, $s \leq k$, for the first k states of

$\gamma(I, pz) \in \mathcal{M} \setminus (\mathcal{M}_{\mathcal{I}} \cup \mathcal{M}_{\mathcal{F}})$.

Nature, and 0 for the remaining states. [9]

For optimal values of the second period consumption q_1 and output levels z, the optimal level of financial market participation and crop insurance is given by

$$\begin{aligned} h_{F+\gamma} =& (A_{F+\gamma}^T A_{F+\gamma})^{-1} A_{F+\gamma}^T (q_1 - pz + \tilde{pz}) \\ =& P_{F+\gamma}(q_1 - pz + \tilde{pz}) \end{aligned}$$

where $h_{F+\gamma} = [h_F, \alpha]$, $A_{F+\gamma} = [A_F, \tilde{I}]$. After substituting for $h_{F+\gamma}$ in the first period of (20), the farmer solves

$$\max_{q_1} \left\{ W\Big(\omega - v_{F+\gamma}^T P_{F+\gamma} q_1 + \Pi(p, w, v_{F+\gamma}^T P_{F+\gamma}), q_1\Big) \right\} \tag{21}$$

and

$$\Pi(p, w, v_{F+\gamma}^T P_{F+\gamma}) = \max_z \left\{ v_{F+\gamma}^T P_{F+\gamma}(pz - \tilde{pz}) + v_F^T P_F \tilde{pz} - c(w, z) \right\}$$

It is apparent from the profit maximization that while the optimal output choice z is independent of the farmer's risk preferences, it depends on the new discount factor $v_{F+\gamma}^T P_{F+\gamma}$. Because the crop insurance can not be replicated in the farmer's financial span, $\mathcal{M}_{\mathcal{F}}$, the distortionary effects of the insurance alter both production choices, like in Chambers (2018), and the farmer's marginal risk attitudes. Furthermore, the optimal consumption vector is determined by the new discount factor $v_{F+\gamma}^T P_{F+\gamma}$.

Assuming preferences and the cost function are differentiable, the first order conditions, at interior, are

$$\frac{\partial W(q_0, q_1)/\partial q_{1s}}{\partial W(q_0, q_1)/\partial q_0} = v_{F+\gamma}^T P_{F+\gamma s}, \ \forall s \in \Omega \tag{22}$$

[9]We assumed that the subsidy $a(I, pz)$ just covers the difference between the market price of insurance, $v_{\mathcal{I}}^T h_{\mathcal{I}}^\star$, and farmer's willingness to pay for it, $v_F^T P_F \gamma(I, pz)$. The case $v_F^T P_F \gamma(I, pz) + a(I, pz) > v_{\mathcal{I}}^T h_{\mathcal{I}}^\star$ is treated similarly.

$$\frac{\partial c(w,z)}{\partial z_s} = v_{F+\gamma}^T P_{F+\gamma s} p_s, \; \forall s \in \Omega, s > k \tag{23}$$

$$\frac{\partial c(w,z)}{\partial z_s} = v_F^T P_{Fs} p_s, \; \forall s \in \Omega, s \leq k \tag{24}$$

The system of equations (22) confirms that the optimal consumption vector is determined by the discount factor $v_{F+\gamma}^T P_{F+\gamma}$. Specifically, the marginal rate of substitution between the stochastic consumption in the second period and the consumption in the first period equals the new discount factor. Separation between consumption and production decisions is shown by the system of equations (23) and (24). According to (23), in the states of Nature where agricultural revenue is higher than the insurance coverage, the optimal output is determined by the discount factor $v_{F+\gamma}^T P_{F+\gamma}$. From (24), the marginal cost of producing in the states of Nature where the insurance is triggered equals the farmer's previous discount factor times the price, $v_F^T P_{Fs} p_s$. This is consistent with the behavior of rational producers that eliminate any arbitrage between the insurance and the existing alternatives in their market span. In this case, the decision to purchase a coverage that is triggered, $I > p_s z_s$, must equal at the margin the benefit of its alternatives in $\mathcal{M}_{\mathcal{F}}$ as represented by the discount factor $v_F^T P_F$. Furthermore, from the insurance pricing equation $v_F^T P_F \gamma(I, pz) + a(I, pz) = v_{\mathcal{I}}^T h_{\mathcal{I}}^{\star}$, it follows $v_F^T P_{Fs} = \partial a(I, pz)/\partial z_s$. In words, the marginal decision to purchase insurance, as given by the change in the subsidy, equals the discount factor $v_F^T P_{Fs}$ for any $s \leq k$.

Hence, in the presence of incomplete financial markets where producers and insurers have access to different closed subspaces of the market span, the subsidized crop insurance affects the farmers' production decisions in two ways. First, the crop insurance changes the discount factor used by the farmers to price, at the margin, the agricultural output. Second, the crop insurance changes the optimal output across states of Nature. However, these alterations are not due to changes in the risk preferences by the farmers, separation between production and consumption still holds, but due to different opportunities made available to them. As rational agents, farmers will not forgo opportunities to raise profits and the new opportunities introduced by the crop insurance can not be ignored. Whether this is a reason for the crop insurance to be subsidized is an entirely different matter.

2 Conclusion

In this paper, we analyzes the optimal behavior of farmers in the presence of incomplete financial markets and a subsided crop insurance. The set up of the problem is that of a farmer with access to a subsidized crop insurance in addition to other non-farm risk mitigating tools. However, there is no *a priori* reason to assume that farmers and potential insurers have access to the same non-farm risk mitigation alternatives. Hence optimal decisions regarding purchasing or selling crop insurance is bound to be influences by the respective set of risk mitigating alternatives.

In this context, we show that crop insurance choices are motivated by profit maximizing behavior and not risk mitigation considerations. However, because farmers and insurers have access to different subsets of the market span, their marginal risk valuations differ. The subsidized crop insurance will enrich the farmers' market span altering the marginal risk valuation, in addition to changing the relative returns across states of Nature.

From a policy perspective, this paper helps fill the gap between supporters and critics of the subsidized crop insurance. By subsidizing the crop insurance, the government expands the farmers' risk mitigating set of alternatives at the cost of influencing their production decisions but not their risk preferences. Whether this is a reason for the crop insurance to be subsidized is an entirely different matter.

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