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Abstract

This paper studies the marginal product of private capital (MPK) with new data and a new framework to obtain a better understanding of international capital allocations and the Lucas Paradox (LP). Our point of departure is three influential studies of MPK's and, based on the most recently available data, the LP is either sustained, inverted, or rejected. We then introduce three improvements in measuring spot MPK's, and the LP clearly reemerges. While these results are provocative, they may be misleading because they do not recognize the dynamics of the capital accumulation process toward steady-states. We develop and estimate a model that allows us to map spot MPK's into steady-state MPK's. The LP remains; the steady-state MPK's for poor countries is 48% to 77% higher than for rich countries. Four policy implications follow from these estimates. First, there is a great deal of misallocated capital globally: 14% to 21% of the global capital stock. Second, this misallocation is primary due to the difference between country-specific steady-state MPK's and the global MPK that would maximize world output. Third, the benefits of optimally reallocating capital and eliminating the LP are modest: 1.0% to 1.5% of global output or $873 to $1,309 billions of 2019 US dollars. Fourth, the estimates for both misallocation and reallocation depend crucially on the elasticity of substitution between capital and labor. Our empirical work uncovered three new puzzles that have emerged beginning in 1990, 1) the MPK's for both poor and rich countries have been rising sharply, 2) the gap has been widening, and 3) the steady-state MPK's exceed the average spot-MPK's. The later result is inconsistent with the Dynamic Inefficiency, Saving Glut, or Secular Stagnation hypotheses.

JEL-Codes: E220, F200, O110, O570.

Keywords: international capital allocations, Lucas Paradox, cross-country marginal products of capital, macroeconomic analyses of economic development.

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International Capital Allocations
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Is capital allocated efficiently internationally? In a smoothly functioning neoclassical world, private capital flows between locations equalize the marginal products of capital (MPK). However, that view was challenged by Lucas (1990) in a provocative article -- "Why Doesn't Capital Flow from Rich to Poor Countries?" His study highlighted that the MPK for poor countries was much higher than that for rich countries. Such large and sustained differences in MPK's define the Lucas Paradox, which is sharply at odds with private capital's pursuit of profitable investment opportunities and diminishing returns to capital accumulation that are at the core of the neoclassical framework. Lucas original analysis has spawned a large literature modifying the definition of and using better data to compute MPK's. Differing conclusions have been reached about the continued existence of the Lucas Paradox and the extent to which private capital is misallocated internationally.

This study examines international capital allocations between poor and rich countries in terms of MPK's with new data and a new framework that result in a series of new findings.[1] Our point of departure is a reassessment of three influential studies using the latest available country panel data from the Penn World Tables (PWT) and the International Monetary Fund (IMF). Some of the important revisions incorporated in the latest versions of these datasets concern the capital goods deflator and depreciation rates and hence the capital stocks, items that play a large role in quantifying MPK's. That analysis of poor and rich countries is contained in Section I and documents that, across the procedures used in the three studies, the Lucas Paradox is either sustained, inverted, or rejected.

[1] There is also an important branch of the literature on international capital allocations that studies capital flows directly (see the surveys in Banerjee and Duflo (2005), Obstfeld and Taylor (2003), and Reinhardt, Ricci, and Tressel (2013)). Capital flows are an important channel for international capital allocations and the equalization of marginal products of capital, but they are not fully informative about whether capital is allocated efficiently. Capital flows to poor countries may be driven by domestic saving, portfolio balancing by investors in rich countries, or the not infrequent misvaluations in financial markets. An absence of capital flows may be consistent with international capital markets that are either performing very poorly or very well if, in the latter case, marginal products are equal across countries. Our focus on marginal products, while also only one part of the international capital allocation story, allows us to assess the efficiency question directly.

Section II introduces three elements that have been omitted in prior computations of spot MPK's and may affect comparisons between poor and rich countries – aggregation, shadow economic activity and government capital.[2] There is an a priori expectation that excluding any of these factors will increase the MPK gap between poor and rich countries, an expectation borne-out in Section II. Based on averages of the spot MPK's, we find that the Lucas Paradox reemerges and that the spot MPK's for both poor and rich countries exhibit a dramatic and puzzling upward trend since 1990.

While this initial evidence is quite interesting, it must be considered with due caution owing to a potentially serious bias. Section III demonstrates that a proper assessment of capital allocation efficiency requires recognizing the dynamics of the capital accumulation process toward steady-states. For countries with initial capital allocations below (spot MPK's above) their steady-state, the bias is positive. This positive bias might explain the results confirming the Lucas Paradox, even if the steady-state MPK's for poor and rich countries are in fact equal. Moreover, the possibility that steady-states differ among countries needs to be recognized.

These results suggest a more formal modeling approach is needed to control for adjustment dynamics and heterogeneous steady-states. Section IV derives a partial adjustment model that maps data on spot MPK's into estimates of the adjustment speed and the steady-state MPK's. Baseline results are reported in Section V. We find that adjustment speeds are moderately quick and are faster for poor countries. Regarding the core question concerning international allocation of capital, the Lucas Paradox remains even after correcting for adjustment dynamics and heterogeneous steady-states.

Section VI contains four robustness checks that largely confirm the above results: introducing a second lag of the dependent variable, constraining the adjustment parameters for poor and rich countries to be equal, using an alternative definition of poor and rich countries, and examining the role of intellectual capital.

With our estimates of steady-state MPK's in hand, we turn toward estimating the economic impact of internationally misallocated capital. Section VII measures misallocation of the capital stock in terms of differences among the average spot MPK's, country-specific steady-

[2] Spot MPK's are the annual MPK's computed according to the formulas in Section I and Appendix B, and they will be contrasted with country-specific and global steady-state MPK's introduced in Section III.

state MPK's, and a global steady-state MPK, the latter maximizing global output. Capital misallocation is substantial. For example, for the full sample period, 17.7% of the capital stock is misallocated. This figure is due mostly to the difference between country-specific and global steady-states. Our computations also document the critical importance of the elasticity of substitution between capital and labor in affecting quantitative assessments of misallocation.

To what extent can the output loss due to misallocated capital be reversed? Section VIII considers counterfactual reallocations of capital. For the global economy for the full sample period, the benefit of optimally reallocating capital is a modest 1.2% of global output. However, the benefit to poor countries amounts to 4.9% of their pre-reallocation output. Reallocation comes at the expense of rich countries. When the optimal reallocation is constrained so that rich countries as a whole are compensated ex-post for lost output, the benefit to poor countries remains a substantial 3.2% of their pre-allocation output.

Section IX concludes and discusses remaining research questions.

I. Background

This section reviews three very influential studies of the international allocation of capital. Each paper defined the MPK differently, driven by the data that were available at the time the study was conducted. In this section, we reevaluate each study with its definition of the MPK but with the most recently available data from the IMF for the period 1970 to 2014. Comparisons are drawn between poor and rich countries for two sub-periods, 1970 to 1990 and 1991 to 2014. See Appendix A for a list of the 88 countries used in this study: 32 are classified as Rich, 56 as Poor. For the sake of completeness, we also present results for the full sample, 1970 to 2014.

All three of the models are based on a decomposition of a neoclassical production function based on Euler's Theorem of Homogenous Functions. Production (Y) in each country is determined by a constant returns to scale production function (F[.]) depending on productivity (A), labor (L), and three types of capital, private (K^P), government (K^G), and natural (K^N),

$$(1) \qquad Y = A\,F\left[L, K^P, K^G, K^N\right],$$

where, for notational simplicity, country (i) and time indices (t) have been omitted. Applying Euler's Theorem to equation (1), we obtain,

$$(2) \qquad Y = A F_L L + A F_{K^P} K^P + A F_{K^G} K^G + A F_{K^N} K^N ,$$

If labor is paid a wage (w) equal to its marginal product, equation (2) can be rewritten to equate the capital income share (CIS^{P+G+N}) to the marginal products of three types of capital weighted by their respective capital/output ratios,

$$(3) \quad Y - w L = \left(1 - \frac{w L}{Y}\right) Y \rightarrow CIS^{P+G+N} = A F_{K^P} \frac{K^P}{Y} + A F_{K^G} \frac{K^G}{Y} + A F_{K^N} \frac{K^N}{Y} .$$

We assume that the flow of private capital is the operative margin along which capital is allocated internationally and focus on the associated MPK equal to $A F_{K^P}$. Equation (3) highlights the fundamental problem in measuring MPK given the competing claims on CIS^{P+G+N} from the other two types of capital.

A major advance in the analysis of international comparisons was the creation by Summers and Heston (1991) of the Penn World Tables (PWT). This work has been revised several times (Feenstra, Inklaar, and Timmer, 2015) and is currently updated and further developed by the Groningen Growth and Development Centre (2019). The versions have ranged from PWT5 to PWT9. Subsequently, the IMF has used these data as a starting point and added some refinements and extensions (most notably data on government capital) to create an international database that is the primary data source used in the current work.

Descriptive statistics for the capital/output ratio and capital income share for produced capital (i.e., the sum of private and government capital) for poor and rich countries are presented in the panels 1 and 2 of Table 1.[3] (See Appendix B for a glossary and details concerning variable definitions and sources and for the criteria for including countries in the sample.) As expected, the capital/output ratio is relatively higher for the rich countries. However, somewhat surprisingly, especially in the face of the "saving glut" (Bernanke, 2005), this ratio has fallen during the second half of the sample, a fall that is larger for the rich countries. Consistent with the evidence in Karabarbounis and Neiman (2014), Piketty (2014), and others, the CIS's have risen by about 300 basis points. Table 1 also includes data on other variables that will feature

[3] Natural capital is omitted in these calculations because of an insufficient amount of data for its capital stock.

prominently in the computation of MPK's: the output/private capital ratio and its components, the price of output relative to the price of capital and its components.

The Lucas Paradox (LP) is that the MPK's for poor countries are much higher than those for rich countries (Lucas, 1990). Constrained by data availability, Lucas solved the fundamental measurement problem in equation (3) by assuming that the marginal products for private and government capital are equal and that natural capital had no impact. These assumptions led to the following definition of the MPK,

$$(4) \qquad MPK^{1}_{i,t} \equiv CIS^{P+G+N}_{i,t} * \left(Y_{i,t} / K^{P+G}_{i,t}\right).$$

Table 2, panel 1 and Figure 1 document that the most recent data continue to support the LP. The average of spot MPK^{1}'s for poor countries is between 36% to 45% larger than those for rich countries.

In an important contribution, Caselli and Feyrer (CF, 2007) provide compelling evidence that the LP does not exist. They introduce two enhancements. First, with what was then recently released data on natural capital, they are able to compute a more accurate MPK by assuming that marginal products for all three types of capital are equal and allowing for variation in natural capital across countries. Second, they adjust the MPK formula for differences in the relative price of capital ($P^{Y}_{i,t} / P^{K^{P}}_{i,t}$). These enhancements are indicated in equation (5) in boldface,

$$(5) \quad MPK^{2}_{i,t} \equiv CIS^{P+G+N}_{i,t} * \left(Y_{i,t} / K^{P+G}_{i,t}\right) * \left(\mathbf{P^{Y}_{i,t} / P^{K^{P}}_{i,t}}\right) * \left(\mathbf{K^{P@+G@}_{i,t} / (K^{N@}_{i,t} + K^{P@+G@}_{i,t})}\right)$$

where the "@'s" on the capital stocks signifies that they are from a different, somewhat limited data source (see Appendix B for a detailed discussion). Both adjustments will tend to lower MPK's in poor countries relative to rich countries. Indeed, CF find in their original article using PWT 6.1 that the MPK's for poor countries is lower than that for rich countries (CF, Table II, 549-550). The ratio is 0.81. With the most recent data, the results are even more dramatic. As shown in Table 2, panel 2 and Figure 1, the Poor/Rich ratio falls to 0.51 and 0.65 in the earlier and later parts of the sample, respectively.[4] In effect, CF's approach documents an "inverse LP."

A third significant study improves on the CF adjustment for natural capital. Monge-Naranjo, Sánchez, and Santaeulàlia-Llopis (2019, henceforth MSS) undertake the difficult task

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[4] Hereafter, we will omit "respectively" when reporting two or more results that refer to a statement in the immediately surrounding text.

of constructing data to measure directly the income flow attributable to natural capital (CIS^{N}). Thus, with respect to natural capital, they are able to address an important aspect of the fundamental measurement problem directly, replacing $A F_{K^N} K^{N} / Y$ in equation (3) by CIS^{N},

$$(6) \qquad MPK^{3} \equiv \left(CIS_{i,t}^{P+G+N} - CIS_{i,t}^{N}\right) * \left(Y_{i,t} / K_{i,t}^{P+G}\right) * \left(P_{i,t}^{Y} / P_{i,t}^{K^{P}}\right).$$

Their initial evidence was consistent with an LP that was declining over time. Their study, however, was based on PWT8 and ended in 2005.[5] When their model is reevaluated with the more recent IMF data and with data through 2014, the results differ. As shown in Table 2, panel 3 and Figure 1, the Poor/Rich ratios are 0.96 and 1.08 in the earlier and later periods. These updated estimates of spot MPK's suggest approximate MPK equalization across countries.

The above results from the models used in three important studies are summarized in the plots at the bottom right in Figure 1 in terms of the Poor/Rich ratios. This re-evaluation with the most recently available data either reaffirms, "inverts," or rejects the LP.

II. New Data And New Adjustments: Aggregation, The Shadow Economy And Government Capital

This section introduces three new adjustments affecting the relative MPK's between poor and rich countries and with which the LP clearly reemerges. We build on the frameworks developed by Caselli and Feyrer (2007) and Monge-Naranjo, Sánchez, and Santaeulàlia-Llopis (2019).

First, in computing the MPK's for poor and rich countries, it is important to keep in mind that there is a great deal of variability in country size that must be accounted for in aggregating the spot MPK's into poor and rich groups. Thus we replace the uniformly-weighted mean spot MPK's reported in Table 2 with capital-weighted means in Table 3, panels 1, 2, and 3. This change tends to lower the means for both poor and rich countries.[6] The impact tends to be greater on rich countries, and the Poor/Rich ratios rise for MPK^{1}, MPK^{2}, and MPK^{3} for all three periods.

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[5] The different datasets have important effects on MPK^{3}. For the period 1970 to 2005 (the final year for which the MSS capital income data are available), Poor/Rich ratios are 1.28 and 1.26 for the PWT8 and PWT9 data, but drop sharply with the IMF data to 0.97. See Appendix C for additional results and alternative data needed to extend the sample period to 2014.

[6] The exceptions are the entries for poor countries for MPK^{2} and for MPK^{3} in the earlier period. Regarding CF's MPK^{2} measure, the Poor/Rich ratios are 0.73 and 0.85, close to their original estimate of 0.81.

Our second adjustment focuses on economic activity that is outside the market and hence not captured by GDP. This adjustment is based on data from Schneider, Buehn, and Montenegro (2010) for legal, non-market activity:

> The shadow economy includes all market-based legal production of goods and services that are deliberately concealed from public authorities for any of the following reasons: (1) to avoid payment of income, value added or other taxes, (2) to avoid payment of social security contributions, (3) to avoid having to meet certain legal labor market standards, such as minimum wages, maximum working hours, safety standards, etc., and (4) to avoid complying with certain administrative procedures, such as completing statistical questionnaires or other administrative forms.

The share of economic activity in the shadow economy is twice as large for poor countries (Table 1, panel 9), and thus this adjustment will have a first-order impact on the pattern of MPK's studied in this paper. The MPK's are modified by multiplying the GDP data by one plus the share of shadow economic activity (SE, measured as a percentage of reported GDP),

$$(7) \qquad MPK_{i,t}^{4} \equiv \left(CIS_{i,t}^{P+G+N} - CIS_{i,t}^{N}\right) * \left(Y_{i,t} / K_{i,t}^{P+G}\right) * \left(P_{i,t}^{Y} / P_{i,t}^{K^{P}}\right) * \left(1 + \mathbf{SE_{i,t}}\right).$$

The spot MPK's increase by approximately 30% for poor countries and 15% for rich countries. A sizeable gap exists between the MPK's for poor and rich countries; the Poor/Rich ratios are 1.37 and 1.47 in the earlier and later periods (Table 3, panel 4 and Figure 2).

The shadow economy adjustment magnifies two important nascent developments. As shown in Figure 2, beginning in 1990, the MPK's for both poor and rich countries begin to rise and the gap between the two groups expands over time. We note in passing that capital controls largely ended in the latter 1980's and the early 1990's saw a dramatic increase in international capital flows (Kaminsky, 2019, p. 1). An exploration of these secular patterns is left for future research. The impact of this finding for the current study is that the analysis should be conducted on two distinct periods – 1970 to 1990 and 1991 to 2014.

The third adjustment recognizes that measures of MPK considered so far mix "apples and oranges," as the income figure represents economic activity from both private and government capital. An important advantage of the IMF data is that it provides separate estimates for each capital stock. Government capital is much more important and much more volatile in poor countries. The ratio of government to private capital ($RK^{G,P}$) is nearly three times larger in the earlier period for poor countries (Table 1, panel 10). In the later period, the ratio for poor countries falls sharply and is only 1.62 times larger than for rich countries. For both groups, the $RK^{G,P}$ ratio has fallen over time. While these capital stock data are an important enhancement for generating more accurate measurements, comprehensive data for the marginal product of government capital is lacking. Returning to the Euler representation in equation (3), we assume that the marginal product of government capital is proportional to that for private capital for poor and rich countries (φ^g, $g = \{poor, rich\}$), and write the MPK as follows,[7]

$$(8) \quad MPK^5_{i,t} \equiv \left(CIS^{P+G+N}_{i,t} - CIS^N_{i,t}\right) * \left(Y_{i,t} / K^P_{i,t}\right) * \left(P^Y_{i,t} / P^{K^P}_{i,t}\right) / \left(1 + \varphi^{\mathbf{g}} * \mathbf{RK}^{\mathbf{G,P}}\right).$$

Data for φ^g are taken from Lowe, Papageorgiou, and Perez-Sebastian (2012, Table 1); $\varphi^{Poor} = 1.2$ and $\varphi^{Rich} = 1.9$ (see Appendix B for details on data construction). The government capital adjustment lowers all spot MPK's. The impact differs between the two groups of countries due to variation in $RK^{G,P}$ and φ^g. As shown in panel 5 of Table 3 and Figure 2, the Poor/Rich ratio rises by about 20%.

Lastly, the adjustments for the shadow economy and government capital are combined in the definition of MPK^6,

$$(9) \quad MPK^6_{i,t} \equiv \left(CIS^{P+G+N}_{i,t} - CIS^N_{i,t}\right) * \left(Y_{i,t} / K^P_{i,t}\right) * \left(P^Y_{i,t} / P^{K^P}_{i,t}\right) * \left(1 + \mathbf{SE}\right) / \left(1 + \varphi^{\mathbf{g}} * \mathbf{RK}^{\mathbf{G,P}}\right).$$

Relative to the MPK^3's that did not reflect these two adjustments, the combined adjustments raise MPK^6 for poor countries and slightly lower it for rich countries. The Poor/Rich ratios are 1.65 and 1.73 in the earlier and later periods (Table 3, panel 6 and Figure 2). The LP clearly reemerges. Moreover, a comparison of the bottom right charts in Figures 1 and 2 highlights that the three adjustments affect the evolution of the Poor/Rich ratios over time. With these three adjustments, introduced either singularly or collectively, the ratio falls in the earlier period and then begins to rise sharply in 1990 and throughout the later period.

[7] This adjustment separating government and private capital parallels the adjustment by Caselli and Feyrer separating natural from produced (government plus private) capital.

III. Bias Due To Transition Dynamics

While the above results are interesting, these spot MPK's may not be fully informative. In evaluating capital allocations, we are interested in the steady-state MPK and how quickly an economy is moving towards this direct measure of capital allocation/misallocation. Even if the steady-states for poor and rich countries are equal, dynamics of the capital accumulation process from initial conditions toward steady-states distort the mapping of spot MPK's into the steady-state MPK's. As we shall discuss in the next section, transition speeds depend on, *inter alia*, the costs of misallocation, adjustment, and finance. Different transition speeds, as well as different initial conditions, are consistent with average spot MPK's showing an LP even if the steady-states are in fact equal.

The possibility of a quantitatively important bias is documented by the hypothetical relationships between observed MPK data and the unobserved steady-state in Figures 3.A and 3.B. The data are generated by a deterministic partial adjustment model; see table notes for details. Figure 3.A is based on the assumptions that the initial conditions and ultimate steady-states are identical for poor and rich countries but that the transition speed to the steady-state is relatively slower for poor countries ($\lambda^{\text{poor}} = 0.10 < \lambda^{\text{rich}} = 0.25$). Using average spot MPK's as a measure of steady-state capital allocation leads to a bias that can be misleading. In this case, the Poor/Rich ratio computed from average spot MPK's is 1.33. The bias disappears as T gets large but, even over a 20-year interval, the bias can be substantial.

The analysis in Figure 3.B reveals a similar bias, though in this case the differentiating factor between poor and rich countries is the initial conditions. For countries approaching the steady-state from above by accumulating more capital, the bias is positive. Even with equal transition speeds, estimates of the steady-state based on spot MPK averages overstate the true value. Again, the Poor/Rich ratio computed from average spot MPK's is 1.33, thus suggesting an LP when none exists.

The important point to be drawn from the above analyses is that an averaging procedure of spot MPK's does not give due allowance to transition dynamics. To support more accurate inferences about capital allocations, steady-states, and transitions thereto we need to rely on the formal model developed in the next section.

IV. New Framework

A. Derivation Of The Estimating Equation

Our estimating equation is derived from an explicit optimization problem. It is considered "semi-structural" because the choice variables are not the policy variables per se. Rather, the means by which policymakers influence the MPK is left in the background, and we assume that policymakers directly choose MPK's. While a more structural model could be easily constructed, it is not easy to estimate. The semi-structural approach pursued here allows us to derive explicitly a linear estimating equation that will prove useful in pursuing our objective of generating an unbiased estimate of the steady-state MPK that accounts for transition dynamics.

We rely on a partial adjustment model developed in the literature in several places, staring with Eisner and Strotz (1959) and Lucas (1967). Our formulation closely follows the derivation in Kennan (1979), though the policymaker's problem is defined in disaggregate terms over a set of industries within a country.

Policymakers choose MPK's to minimize costs. Costs arise from misallocation and adjustment for country i for each of its j industries. Misallocation costs occur because the current MPK deviates from MPK*. These deviations are squared and then multiplied by a coefficient, ζ^M, that translates squared deviations into pecuniary costs. The second cost arises from the adjustment of the MPK's (and the underlying capital stocks). These costs represent lost output from disruptions to the existing production process as the MPK is altered and capital allocated. These installation and "teething" costs are a standard element in modeling input demands (see the surveys by Chirinko (1993), Caballero (1999), and Bond and van Reenen (2007)). Adjustment costs are modeled as the change in the MPK, squared, and then multiplied by a coefficient, ζ^A, that translates squared changes into pecuniary costs.

Misallocation and adjustment costs are embedded in the following dynamic cost minimization problem for the j^{th} industry in country i, discounted over time t by a constant discount factor (R), and summed across industries,

$$(10)\; L_{i,j} = \underset{\{MPK_{i,j,t}\}}{MIN} \sum_{t=1}^{\infty} R^t \left\{ \zeta^M \left(MPK_{i,j,t} - MPK^*_{i,j} \right)^2 + (\zeta^A/2)\left(MPK_{i,j,t} - MPK_{i,j,t-1} \right)^2 \right\} wt_{i,j,t}$$

where $R \equiv (1+r)^{-1} < 1\ (0 < r < 1)$ and $wt_{i,j,t}$ is a fixed weight representing the relative importance of industry j in country i at time t. Optimal choices of the MPK lead to the well-known partial adjustment model. Differentiating equation (10) with respect to $MPK_{i,j,t}$ and rearranging, we obtain the following equation,

$$(11a)\quad \Delta MPK_{i,j,t}\, wt_{i,j,t} = \lambda^{g}\left(MPK^{*}_{i,j} - MPK_{i,j,t-1}\right) wt_{i,j,t} = \left(\alpha_{i,j} - \lambda^{g} MPK_{i,j,t-1}\right) wt_{i,j,t}$$

$$(11b)\quad \alpha_{i,j} \equiv \lambda^{g} MPK^{*}_{i,j}$$

where λ^{g} is the stable root solving the second-order difference equation generated by the first-order conditions and determines the speed of adjustment to the steady-state. Summing across the J industries and defining the aggregates as weighted averages of the industry components, we obtain the following equation for the aggregate MPK's for a given country,

$$(12)\quad \Delta MPK_{i,t} = \alpha_{i} - \lambda^{g} MPK_{i,t-1}.$$

The econometric estimates are based on a panel of all countries, and equation (12) is supplemented with time fixed effects (τ_t) and a stochastic error term ($\varepsilon_{i,t}$),

$$(13)\quad \Delta MPK_{i,t} = \alpha_{i} - \lambda^{g} MPK_{i,t-1} + \tau_t + \varepsilon_{i,t},$$

where the adjustment speed parameter (λ^{g}) varies by poor and rich groups of countries. As is common in the literature starting with at least Sims (1974), the model variables are specified as logs. Thus, percentage changes in the MPK are proportional to the percentage difference between the steady-state and current MPK's.

B. Adjustment Speed (λ^{g})

The adjustment speed parameter is a complicated function of the primitives in the optimization problem – ζ^{M}, ζ^{A}, and R,

$$(14)\quad \lambda^{g} = \lambda\left[\zeta^{M}, \zeta^{A}, R[r]\right] = 1 - \left\{ \frac{(1 + R + \zeta^{M}/\zeta^{A}) - \sqrt{(1 + R + \zeta^{M}/\zeta^{A})^{2} - 4R}}{2R} \right\}.$$

Differentiation of equation (14) with respect to each of the arguments and some tedious manipulations yield some interesting insights:

- $\lambda^g_{\zeta^M} > 0$ implies that the higher the cost of misallocation, the faster countries will converge to the long-run desired value because, ceteris paribus, they wish to eliminate losses as quickly as possible.
- $\lambda^g_{\zeta^A} < 0$ implies the obvious result that, with larger adjustment costs, the approach to the steady-state is slower.
- $\lambda^g_r < 0$ implies the higher the interest rate (or the lower the discount factor), the slower adjustment. Higher r could reflect finance constraints. With higher interest rates, the discounted costs of misallocation are lower, so they are eliminated more slowly by optimizing countries. In a sense, this result is the mirror image of that for ζ^M above.

C. Country-Specific Steady-State MPK (MPK_i^*)

An appealing property of the partial adjustment specification is that it readily yields an estimate of the country-specific steady-state MPK in terms of estimated parameters. The steady-state is defined by $MPK_{i,t} = MPK_{i,t-1}$, $\varepsilon_{i,t} = 0$, and $\phi_t = 0 \ \forall t$. The latter assumption effectively treats the ϕ_t's as incidental parameters that we remove from α_i by subtracting the mean of the time fixed effects ($\bar{\phi}$). With these restrictions, logarithmic versions of equations (13) and (11b) imply that the steady-state marginal product of capital is as follows,

$$MPK_i^* = EXP\left[\frac{\alpha_i[\lambda^g] + \bar{\phi}}{\lambda^g}\right], \tag{15}$$

where $\bar{\phi}$ is the average of the time fixed effects and the dependence of $\alpha_i[\lambda_g]$ on λ_g (through the second term on the right-side of equation (13)) is indicated explicitly.[8]

D. Correcting For The Nickell Bias

In panel models with country-specific effects and a "small" number of observations in the time dimension, the coefficient on the lagged dependent variable is biased. The conventional analysis of this bias is conducted in terms of an AR(1) model in which the positive coefficient on the lagged dependent variable is biased downward. In our first-difference model, this effect will result in an upwardly biased estimate of λ^g. One solution to this problem is to transform the

[8] In the two-way fixed effects model, one time fixed effect (ϕ_t) is dropped to avoid singularity during estimation. The $\bar{\phi}$ term is included in (15) so that the computation MPK_i^* is invariant to which time fixed effect is dropped.

model appropriately and then find suitable instruments. We do not follow this approach because of the general difficulty of finding suitable instruments, a challenge that is even more daunting in our panel of 88 countries for 45 years. Instead, we exploit the simple structure of our model that excludes additional endogenous regressors and correct the bias directly with the formula developed by Nickell (1981) in his seminal article (see the Table 4 note for details). Correcting for the Nickell bias has a very small impact on our estimates of the steady-state MPK because λ^g has a similar effect on both the numerator and denominator of MPK_i^* in equation (15).

V. Baseline Results

Panel 1 of Table 4 contains our baseline results from estimating equation (13) for $MPK_{i,t}^6$ (equation (9)) and computing the steady-state MPK from equation (15). There are three noteworthy findings. First, the LP continues to be in evidence. The Poor/Rich ratio of steady-state MPK's is 1.48 and 1.77 in the earlier and later periods.

Second, adjustment speeds vary among poor and rich countries. In the earlier period, the half-lives (indicated in the table by "/../") are short for poor countries (2.1 years) and much longer for rich countries (5.8 years). In the later period, adjustment speeds are slower and closer together; the comparable half-lives are 3.5 and 6.6 years. In either period, poor countries move toward their steady-states more quickly than rich countries. While we do not explore this result further, we note that it is consistent with greater misallocation costs in our formal model (cf. the derivative of λ^g with respect to ζ^M in equation (14)).

Third, the relations between the steady-state and average spot MPK's differ by period. In the earlier period, there is very little distance between these two MPK's (-110 and 50 basis points for poor and rich countries). This result suggests some combination of adjustment speeds that are sufficiently quick and MPK shocks that are sufficiently small so that, from 1970 to 1990, economies did not stray too far from their steady-states.

However, in the later period, the increases in the steady-state MPK's for both poor and rich countries are substantial and exceed those for the average spot MPK's. The steady-state MPK's are now much greater than the average spot MPK's: 400 and 190 basis points for poor and rich countries. This result suggests an over-accumulation of capital and, as both poor and rich countries transit to their steady-states, their MPK's need to rise through reductions in capital/output ratios. Thus, the question is not "why doesn't capital flow from rich to poor countries" but rather "why has capital flowed so freely to both poor and rich countries in recent years?"

One answer to this later question is a global saving glut (Bernanke, 2005). In this case, the excess amount of saving and subsequent capital accumulation lowers the spot MPK's relative to steady-state values. The possibility of over-accumulation has been established rigorously by Diamond (1965). In this situation, the economy is labelled dynamically inefficient because the capital sector is reducing consumption and welfare could be increased by "eating" some of current capital or failing to replace depreciating capital. Our finding that average spot MPK's are below steady-state MPK's suggests that economies may have been dynamically inefficient in the later period. The Abel, Mankiw, Summers, and Zeckhauser (AMSZ, 1989) test for dynamic inefficiency is whether outflows from the capital sector in terms of capital income are less than inflows into the capital sector in terms of gross capital accumulation. Is the capital sector a spout (efficient) or a sink (inefficient) (AMSZ, p. 2)? In our notation, outflows for country i at time t are measured by private MPK and inflows by private gross capital formation, each stated per unit of private capital,

$$\text{(16a)} \qquad OUTFLOW_{i,t} \equiv MPK^{P}_{i,t},$$

$$\text{(16b)} \qquad INFLOW_{i,t} \equiv \Delta K^{P}_{i,t} / K^{P}_{i,t-1} + \delta^{P}_{i,t}$$

where $\delta^{P}_{i,t}$ is the depreciation rate for private capital. The dynamic inefficiency hypothesis is evaluated by capital-weighted averages of equations (16a) and (16b) for poor and rich countries,

$$\text{(16c)} \qquad OUTFLOW_{g,t} = \sum_{i \in g} OUTFLOW_{i,t} \; (K^{P}_{i,t} \;/ \sum_{i \in g} K^{P}_{i,t}),$$

$$\text{(16d)} \qquad INFLOW_{g,t} = \sum_{i \in g} INFLOW_{i,t} \; (K^{P}_{i,t} \;/ \sum_{i \in g} K^{P}_{i,t}),$$

$$\text{(16e)} \qquad OUTFLOW_{g,t} \; >=< \; INFLOW_{g,t}. \qquad g=\{poor, rich\}$$

The results of the test for poor and rich countries are presented in Figure 4. Outflows have clearly exceeded inflows, and the gap between the two has been growing. Based on the AMSZ test, we conclude that economies are not dynamically inefficient.

Figure 4 also rejects the implication of the Saving Glut hypothesis that capital formation has been rising, as INFLOW's trend downward in the later period. By contrast, the Secular

Stagnation hypothesis (Summers, 2015) predicts a lower rate of capital formation. However, this theory is at odds with rising MPK's shown as an increase in OUTFLOW's in Figure 4. We leave it to future work to get a better understanding of the forces affecting MPK's and capital formation since 1990.

VI. Robustness

This section examines the robustness of our baseline results in four dimensions. All results in Table 5 are based on MPK^6 and should be compared to the baseline results for steady-state MPK^6 in panel 1 of Table 4.

First, we add a second lagged dependent variable to equation (13). While this specification does not follow from the formal derivation, it can nonetheless be useful in capturing any dynamics that may have been missed in our single lagged specification. The results in the panel 1 of Table 5 confirm that there is very little change in the λ^{Poor} and λ^{Rich} (computed as the sum of the two coefficients on the lagged dependent variables), the MPK*'s, or the Poor/Rich ratios.

Second, we examine the extent to which our estimates of the Poor/Rich ratio are due to different adjustment speeds between poor and rich countries by reestimating our baseline model constraining λ^{Poor} to equal λ^{Rich}. The results in panel 2 of Table 5 show that the constrained estimate is close to the unconstrained estimate for λ^{Poor}, which is consistent with the greater number of poor countries in our sample and the greater variation in their MPK's. There are very modest impacts of this constraint on the Poor/Rich ratio and the MPK*'s for both poor and rich countries. The latter result may be surprising, but it follows because λ enters both the numerator and denominator of the equation determining MPK* (cf. equation (15)).

Third, the classification of countries into poor and rich groups is based on the four-way World Bank country classification (by income) – high, upper middle, lower middle and low. In our baseline results, we define the rich and poor groups as follows: rich = {high}, poor = {upper middle, lower middle, low }. To examine the sensitivity of our results to this classification, we move the upper middle countries from the poor group to the rich group. As shown in panel 3 of Table 5, this alternative classification has a modest impact on reducing the Poor/Rich ratios from 1.48 to 1.27 in the earlier period and 1.77 to 1.70 in the later period.

Fourth and last, a potentially important omission in our dataset is intellectual capital, whose role has clearly become more prominent in recent years. This omission implies that the capital stocks entering the denominator of our MPK equation (cf. equation (9)) are too small, and hence our MPK estimates are too large. However, since intellectual capital has had more impact in rich countries, this upward bias is likely greater for rich countries. Thus, the reported Poor/Rich ratios are biased downward, and the omission of intellectual capital makes it more difficult to obtain results consistent with the LP.

Moreover, the impact of intellectual capital is modest. While there has been a great deal of investment, especially in recent years, the depreciation rate is higher for intellectual capital.[9] To understand how much our results might be affected, we use U.S. data to estimate the intellectual capital stock (see the ICA_t^K variable in Appendix B). The ratio of intellectual capital to private fixed capital is very small, 3.7% and 6.5% for the earlier and later periods. If these figures are applied only to the steady-state MPK's for rich countries, the Poor/Rich ratios in panel 1 of Table 4 for steady-state MPK^6 rise from 1.48 to 1.54 in the earlier period and 1.77 to 1.88 in the later period.

VII. Misallocation Of Private Capital

We now quantify some of the policy implications associated with our estimates in this and the next section. We focus on the misallocation of private capital that arises among the average of the spot MPK's ($\overline{MPK}_i^{SPOT}$), the country-specific MPK's (MPK_i^*), and the global MPK (MPK^{**}). The latter defined as the MPK to which all countries would converge if the existing capital stock was allocated to maximize global output. Note that there is no country subscript on MPK^{**}; its construction will be discussed in Section VIII.

[9] These higher depreciation rates might explain the rise the MPKs in the later period (cf. panel 6 of Table 3). However, the depreciation rate for intellectual capital has little effect, as the capital depreciation rate (for all capital stocks) including intellectual capital rises by only 0.50% between the earlier and later periods relative to the capital depreciation rate (for all capital stocks) excluding intellectual capital. See the construction of the ICA_t^D variable in Appendix B for details.

Our computations are based on the first-order condition for capital from a CES production function that relates the capital/output ratio to the MPK and two parameters, a distribution parameter, ϕ, and the elasticity of substitution between labor and capital, σ. (The value of the latter parameter will prove crucial to the results.)

$$\text{(17a)} \qquad \left(K_i^P / Y_i\right) = MPK_i^{-\sigma} \phi^{\sigma} .$$

Taking logs and rearranging, we obtain the following equation that determines the percentage change in the capital/output ratio,

$$\text{(17b)} \qquad dK_i^P / K_i^P - dY_i / Y_i = -\sigma \, dMPK_i / MPK_i = -\sigma \, \alpha_i ,$$

$$\text{(17c)} \qquad \alpha_i \equiv dMPK_i / MPK_i .$$

To isolate the change in the capital stock, we begin with a neoclassical production function, differentiate it with respect to K_i^P, set the resulting marginal product of capital equal to the cost of capital, and define the private capital income share (CIS_i^P) as this cost of capital multiplied by the private capital/output ratio to obtain the following relation,

$$\text{(18)} \qquad dY_i / Y_i = CIS_i^P \, dK_i^P / K_i^P .$$

Using equations (18) to eliminate dY_i / Y_i in equation (17), we can write the percentage change in the private capital stock as follows,

$$\text{(19)} \qquad dK_i^P / K_i^P = \left| \left(-\sigma \, \alpha_i\right) / \left(1 - CIS_i^P\right) \right| ,$$

which is proportional to σ. Since we are interested in the distorting effects of any misallocation of capital, equation (19) is evaluated as an absolute value. The aggregate value is computed as a capital-weighted average of equation (19).

Equation (19) is intuitive. Consider the initiating shock ($-\sigma \, \alpha_i$ in the numerator) that results in only a change in K_i^P, with output constant, so that the new capital/output ratio is attained. At this temporary allocation, an inconsistency exists because Y_i does not reflect the increment to K_i^P. (This delay in adjusting output could be due to gestation lags between when capital is acquired and when it begins to contribute to production.) To resolve this inconsistency, Y_i needs to be incremented according to equation (18) by CIS_i but, in order to maintain the appropriate capital/output ratio, K_i^P needs to be incremented appropriately. Again there is an inconsistency between the capital stock and output that is resolved in the limit and represented in the denominator by $1 - CIS_i$. Convergence of this process is assured by $0 < CIS_i < 1$.

Table 6 reports sizeable misallocations in capital. All results are for MPK^6 and are computed with the same σ for poor and rich countries.[10] We begin our analysis with the results for all countries with no distinction by income level, which we label global in contrast to poor or rich. In the first row, first column of Panel A based on $\sigma = 0.5$, differences between the average spot MPK's and the global steady-state MPK ($\overline{MPK_i}^{SPOT} - MPK^{**}$) indicate that 17.5% of the global private capital stock is misallocated. The remaining two rows in Panel A consider the same differences in MPK's but allow the substitution between capital and labor to rise to 1.0 (the value associated with the all-too-frequently used Cobb-Douglas production function) and 1.5. The results increase dramatically to 35.4% and 53.1%, and document that the value of σ is crucial for evaluating capital misallocations. Based on our recent work (Chirinko and Mallick, 2017) and a survey of the literature (Chirinko, 2008), we believe that the lower figure of 17.5% is most accurate.

Panels B and C decompose these differences. Panel B reports differences between $\overline{MPK_i}^{Spot}$ and MPK_i^*, and the misallocations are quite small. By contrast, Panel C reports the misallocation due to differences between MPK_i^* and MPK^{**}. These estimates of misallocations are much larger. The conclusion to be drawn is that capital misallocation is largely driven by differences between country-specific and the global steady-state MPK's.

Columns 2 and 3 repeat the exercise for the earlier and later periods for all countries. The patterns of results are similar, and misallocation is larger in the later period.

Columns (4) to (9) repeat these exercises for the three periods and for countries divided into poor and rich groups. The patterns evidenced in the global analysis persist. Additionally, misallocation is relatively much larger for poor countries in the earlier period. However, in the later period, the overall misallocation is about equal between the poor and rich groups.

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[10] Mallick (2012) finds that there is only a modest difference in σ's between poor and rich countries: $\sigma^{Poor} = 0.30$ and $\sigma^{Rich} = 0.40$.

While the substantial misallocation of private capital documented in Table 6 is of interest, it does not inform us about the potential benefits of reallocating capital in an optimal fashion. These benefits are quantified in the next section and will be tempered by changes in MPK's as capital moves from low to high MPK countries and by the fixed stock of global capital.

VIII. Reallocation Of Private Capital

This section takes a global perspective and quantifies the additional output that would be forthcoming if the existing capital stock was allocated optimally across all countries. As is well known, the capital allocation that maximizes global output equates the MPK's across all countries to a single MPK, MPK^{**}.

This global optimum MPK is calculated according to an iterative scheme; details are provided in Appendix D. We begin by assuming an arbitrary initial value for MPK^{**}. This initial estimate determines for the first iteration the percentage changes in the MPK's (from the country-specific steady-state MPK's, the MPK_i^*'s) that equate MPK's across all countries. In turn, this percentage change determines the private capital stocks with a discrete version of equation (19) normalized to K_i^P. We then compute the deviation between the sum of these estimated K_i^P and the fixed amount of global private capital and use this deviation to compute a new estimate of the global optimum MPK. The process is repeated until the deviation is close to zero. Given the negative and monotonic relation between MPK's and capital stocks, the process is guaranteed to converge to a unique solution. The change in global output due to this globally optimal allocation of private capital is computed with an output-weighted average of a discrete version of equation (18) with dK_i^P / K_i^P defined by equation (19).

The percentage changes in global output are presented in Table 7, row 1, columns 1 to 3 and range from 1.0% to 1.5%.[11] Our estimate of the change in output increases with the

[11] CF (2007, Table VI) and MSS (2019, Table 6, average of QMPK) estimate capital reallocation benefits of 0.1% and 3.0%. Since these estimates are based on a Cobb-Douglas production function, they are most comparable to those in our Table 7 with $\sigma = 1$ of 2.4%. Our estimate is much larger than that of CF because they found near equalization of MPK's for poor and rich countries and hence little scope for beneficial reallocation. Note that the estimates are not strictly comparable because of differences in spot MPK's for produced capital (CF and MSS) vs. steady-state MPK's for private capital (this study), as well as differences in samples.

elasticity of substitution; with an overly large elasticity of 1.5, the increase in global output is 4.4%. The 1.2% increase is our preferred estimate and represents an increase in output of $1,047 billion (2019 first quarter U.S. dollars) (see Appendix E for details about the dollar figure computations reported in this paragraph). By way of comparison, the fiscal stimulus programs enacted by the United States (ARRA) and China in response to the global financial crisis amounted to $774 and $677 billion (2019 first quarter U.S. dollars).

The remaining entries in panel A separate the change in global output between poor and rich countries. The $\text{MPK}_i^{*\text{Rich}} = 0.144$, which is very close to $\text{MPK}^{**} = 0.156$. This result indicates that the rich countries as a group are fairly close to the globally optimal capital allocation. By contrast, the $\text{MPK}_i^{*\text{Poor}} = 0.235$ is very far from MPK^{**}, and the scope for improvement lies in reallocating capital from rich to poor countries. For $\sigma = 0.5$ for the full sample, output in poor countries increases by 4.9%, while it decreases by -1.1% in rich countries. These figures are consistent with the modest gain in global output discussed above because a disproportionate amount of world capital and world output is concentrated in the rich countries.

The above results may be somewhat unrealistic since it involves the rich countries suffering a loss in output and the rich countries control the international economic organizations that might lead the effort at such a redistribution. A second scenario imposes an additional constraint that the rich countries do not suffer any loss in output due to an ex-post transfer of output from the poor countries under the new MPK** (i.e., a "compensating variation," a concept used frequently in public economic analyzes). In this case, the results in columns 1 to 3 are identical in panels A and B, and the percentage increase in output for poor countries drops drop from 4.9% to 3.2%. Insofar as incremental income has very high utility in poor countries, this change may have meaningful welfare consequences.

Lastly, we examine the impact of the shadow economy and government capital adjustments on reallocations by repeating the analysis in panel A with steady-state MPK_i^3 replacing steady-state MPK_i^6. As shown in panel C, the estimated benefits of optimally reallocating capital are now approximately one-third lower than the values reported in panel A for steady-state MPK_i^6. These lower values are due to the Poor/Rich ratio based on the MPK_i^3's being smaller than the Poor/Rich ratio based on the MPK_i^6's and hence the scope for beneficial reallocations being smaller. The shadow economy and government capital adjustments introduced in Section II matter for quantitative evaluations.

IX. Conclusions And Remaining Research Questions

This paper studied the marginal product of private capital (MPK) with new data and a new framework to obtain a better understanding of international capital allocations. We examined the Lucas Paradox (LP) -- large and sustained differences in the MPK's between poor and rich countries – with country panel data for the period 1970-2014. Our point of departure was three influential studies of MPK's and, based on the most recently available data, the LP was either sustained, inverted, or rejected by the frameworks used in these papers. We then introduced three improvements in measuring spot MPK's -- aggregation, the shadow economy, and government capital -- and found that the LP clearly reemerges.

While these results are provocative, they may be misleading because they do not recognize the dynamics of the capital accumulation process toward steady-states. To control for this bias, we developed and estimated a partial adjustment model that allowed us to map spot MPK's into steady-state MPK's. The LP remained. The steady-state MPK's for poor countries were 48% higher than for rich countries for the earlier period (1970-1990); the comparable statistic for the later period (1991-2014) was 77%.

Four policy implications followed from these estimates. First, there was a great deal of misallocated capital globally. Our preferred estimates ranged from 14% to 21% of the global capital stock. Second, this misallocation was primary due to the difference between country-specific steady-state MPK's and the global MPK that would maximize world output. Third, the benefits of optimally reallocating capital and eliminating the LP were modest. Our preferred estimates ranged from 1.0% to 1.5% of global output or $873 to $1,309 billions of 2019 US dollars. Fourth and finally, the estimates for both misallocation and reallocation were shown to depend crucially on the elasticity of substitution between capital and labor. Our preferred estimate of this elasticity is 0.50. If the computations are based on an elasticity of 1.0 (the value associated with the all-too-frequently used Cobb-Douglas production function), our misallocation and reallocation estimates are doubled.

Our empirical work uncovered three new puzzles. Since 1990, the MPK's for both poor and rich countries have been rising sharply and the gap has been widening (see Figure 2). Future work will explore how, if at all, this finding is related to the simultaneous rise in the capital income share and whether it is linked to rising markups or some other factors affecting income inequality.

We also found that the steady-state MPK's exceeded the average spot-MPK's for the later period, a result suggesting that capital has been over-accumulated in recent years (cf. Tables 3 and 4). Our initial examination (Figure 4) of this puzzle indicated that neither the Dynamic Inefficiency hypothesis of Diamond (1965), the Saving Glut hypothesis of Bernanke (2005), nor the Secular Stagnation hypothesis of Summers (2015) were consistent with this finding. Answering the question "why has capital flowed so freely to both poor and rich countries in recent years?" will be left for future work trying to obtain a better understanding of the forces affecting MPK's and capital formation since 1990.

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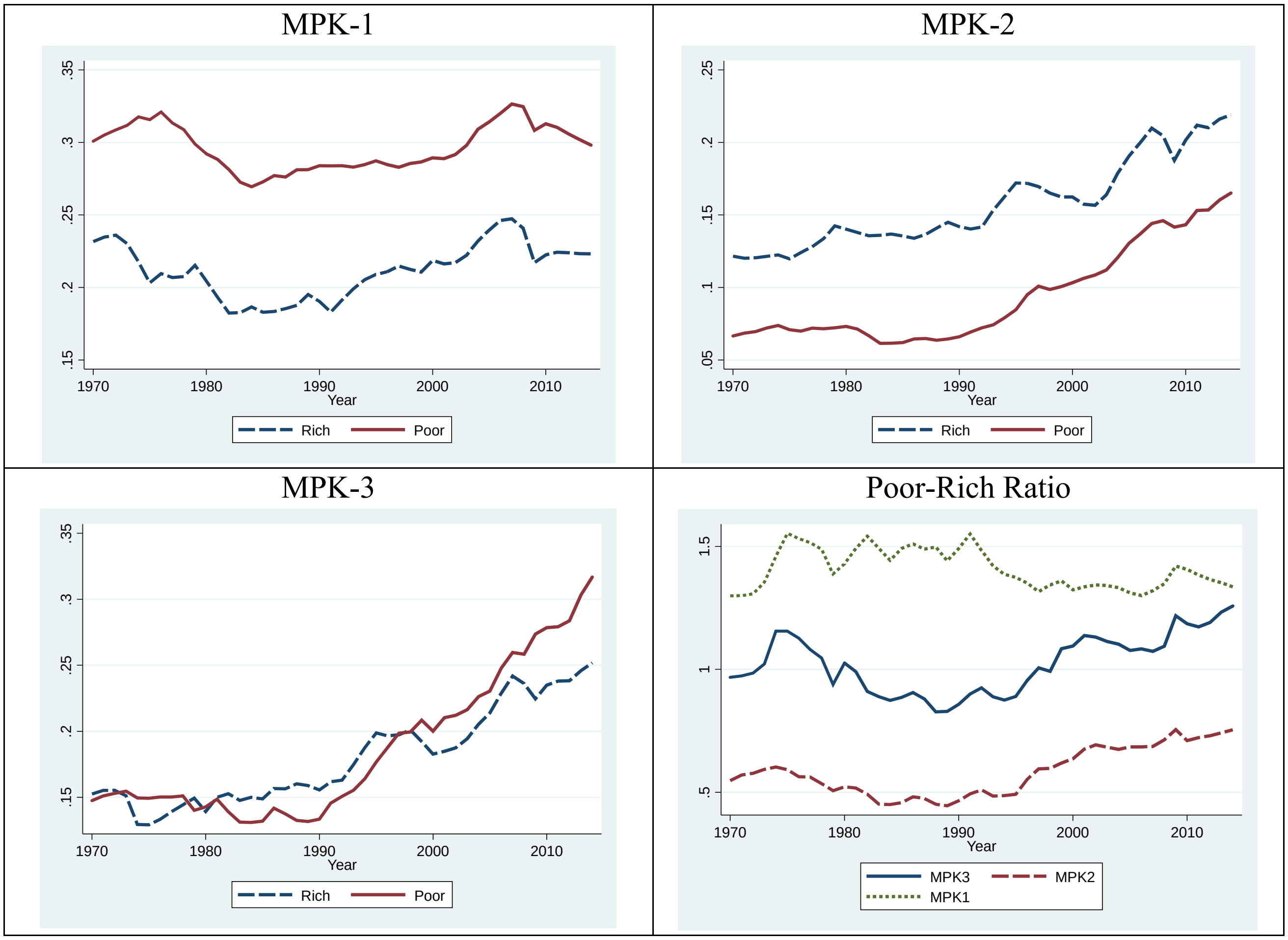
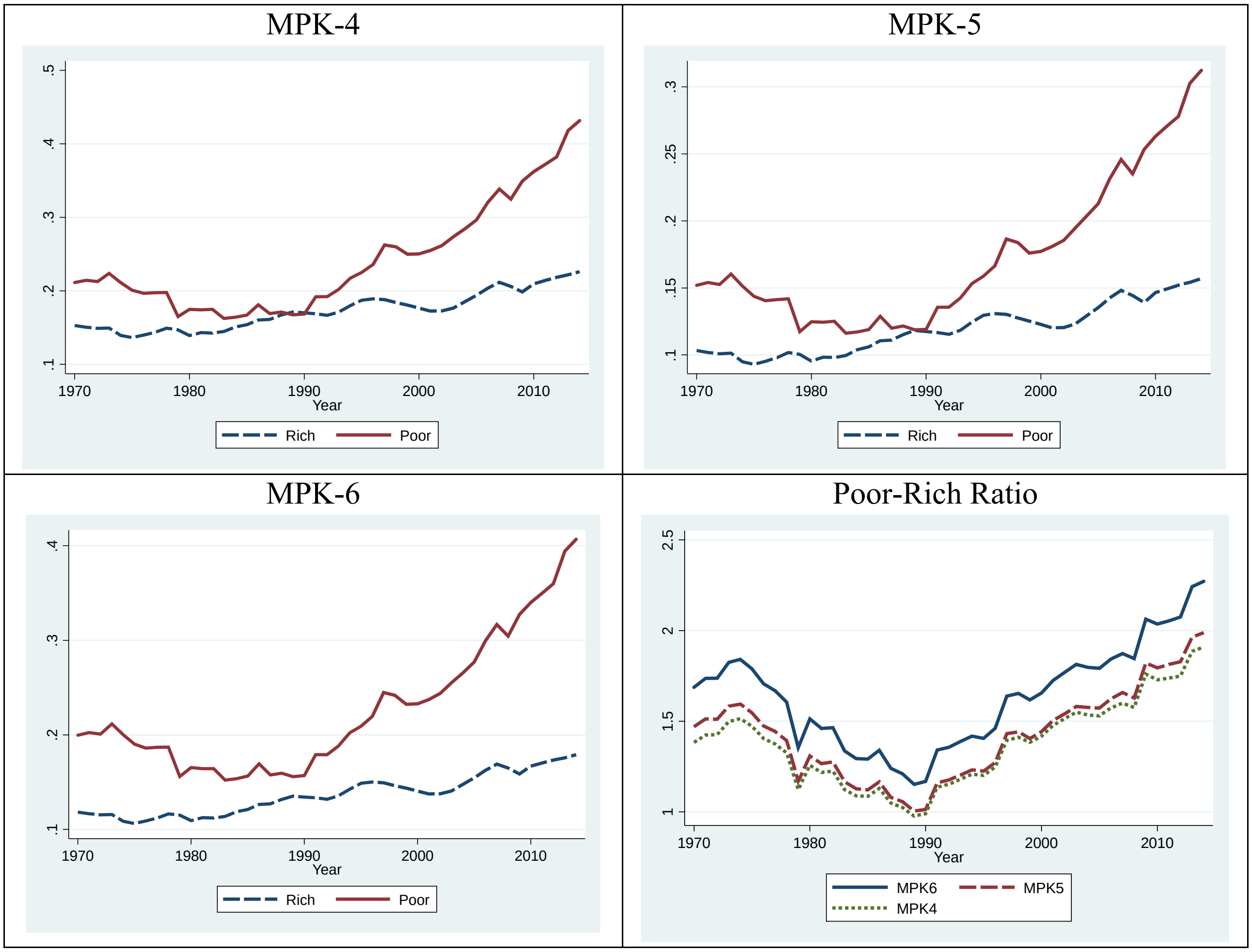
Figure 1: Spot MPK's. Background Uniformly-Weighted Means And Ratios Of Means 1970-2014



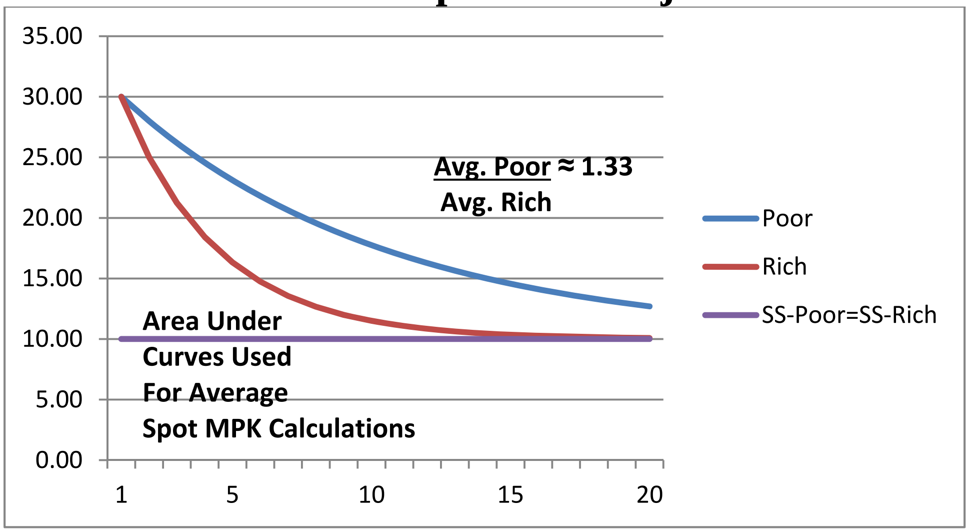
Notes: See the notes to Table 2 for details.

Figure 2: Spot MPK's. Three Adjustments Capital-Weighted Means And Ratios Of Means 1970-2014



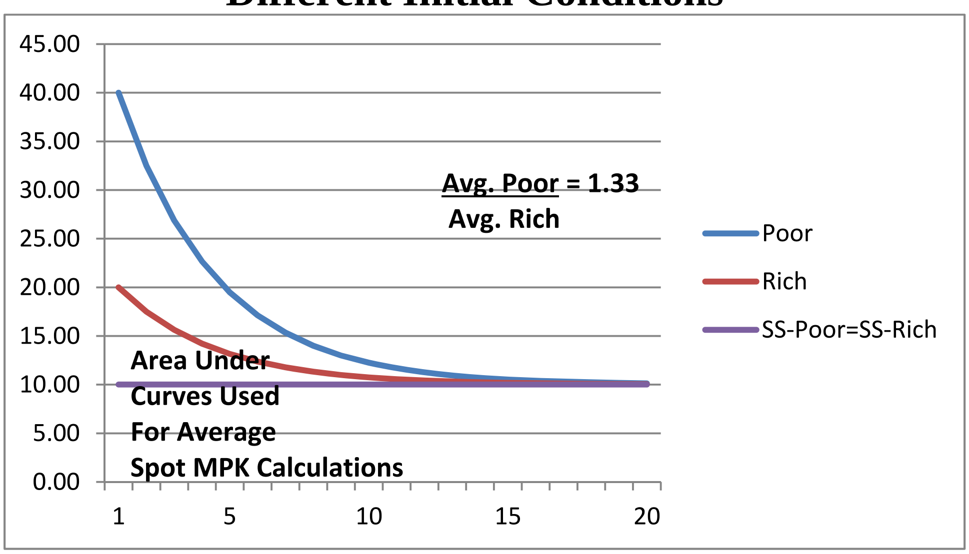
Notes: See the notes to Table 3 for details.

Figure 3.A: Hypothetical Bias In Estimating Steady-State MPK's Different Speeds Of Adjustment



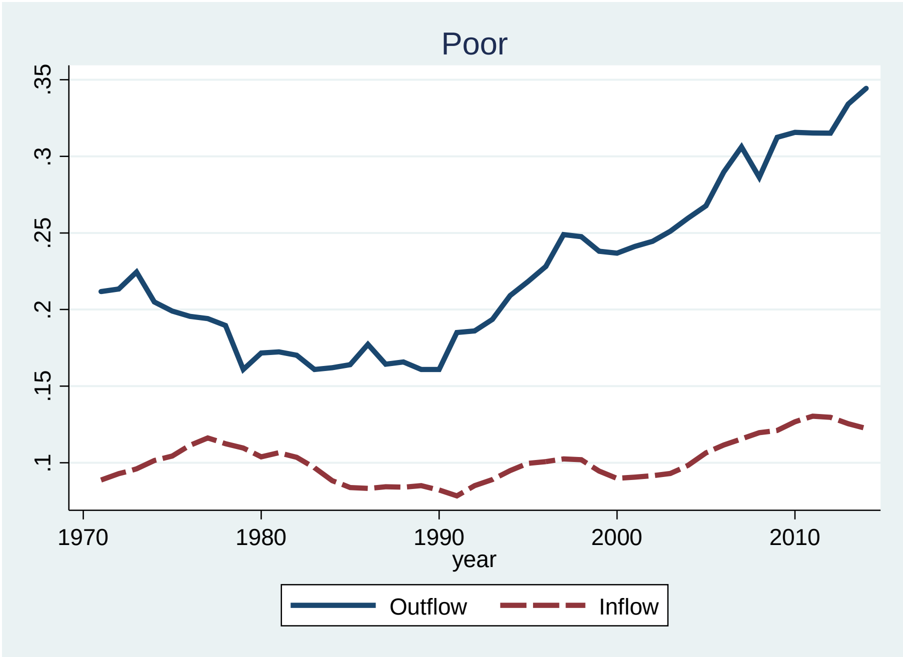
Notes: Hypothetical data are generated by the following partial adjustment model,
$\Delta MPK_t = \lambda^g (MPK^* - MPK_{t-1})$, where $\lambda^{poor} = 0.10$, $\lambda^{rich} = 0.25$,
$MPK_0^g = 30.0$, $MPK^{*g} = 10.0$, $g = \{poor, rich\}$.

Figure 3.B: Hypothetical Bias In Estimating Steady-State MPK's Different Initial Conditions



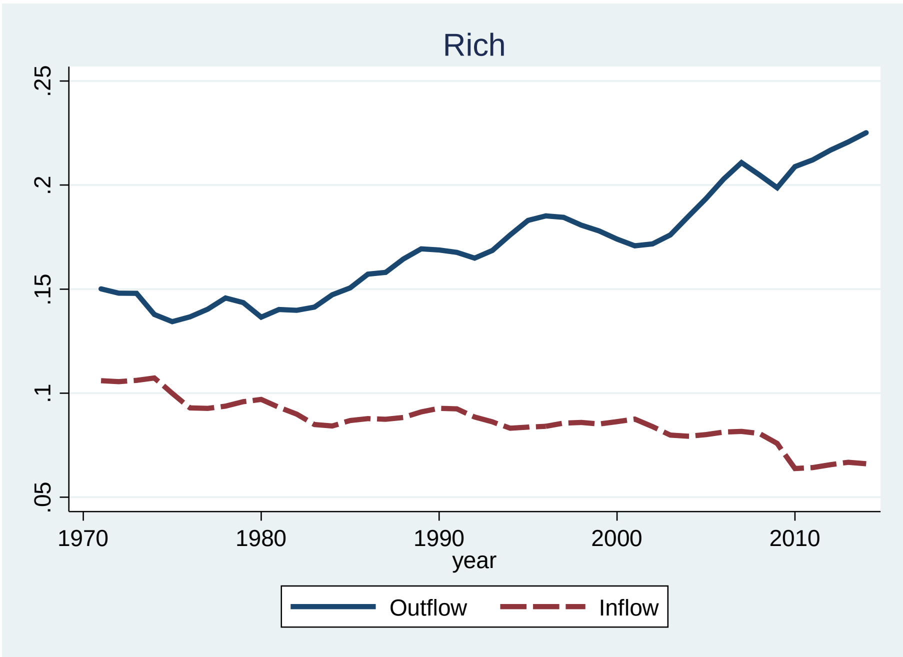
Notes: Hypothetical data are generated by the following partial adjustment model,
$\Delta MPK_t = \lambda^g (MPK^* - MPK_{t-1})$, where $\lambda^{poor} = \lambda^{rich} = 0.25$,
$MPK_0^{poor} = 40.0$, $MPK_0^{rich} = 20.0$, $MPK^{*g} = 10.0$, $g = \{poor, rich\}$.

Figure 4.A: Capital Outflows And Inflows: Poor Countries



Notes: See Section V for a discussion of these results and equations (16) and (17) for the definitions of Outflows and Inflows.

Figure 4.B: Capital Outflows And Inflows: Rich Countries



Notes: See Section V for a discussion of these results and equations (16) and (17) for the definitions of Outflows and Inflows.

Table 1: Descriptive Statistics For Select Variables

	1970-2014	1970-1990	1991-2014
	(1)	(2)	(3)
1. Ratio Of Produced Capital Stocks To Output ($K_{i,t}^{P+G} / Y_{i,t}$)			
Poor	2.151 [2.009] {0.424}	2.220 [1.960] {0.467}	2.090 [2.014] {0.413}
Rich	2.668 [2.480] {0.395}	2.905 [2.780] {0.468}	2.460 [2.340] {0.334}
Poor / Rich	0.806 [0.810]	0.764 [0.705]	0.850 [0.861]
2. Capital Income Share For Produced Capital ($CIS_{i,t}^{P+G}$)			
Poor	0.489 [0.487] {0.233}	0.473 [0.472] {0.260}	0.503 [0.500] {0.223}
Rich	0.450 [0.419] {0.254}	0.434 [0.391] {0.281}	0.465 [0.452] {0.241}
Poor / Rich	1.086 [1.161]	1.090 [1.207]	1.082 [1.105]
3. Output/Private Capital Ratio ($Y_{i,t} / K_{i,t}^{P}$)			
Poor	1.168 [0.840] {0.893}	1.309 [0.795] {1.345}	1.045 [0.871] {0.586}
Rich	0.750 [0.543] {0.993}	0.790 [0.491] {1.403}	0.715 [0.569] {0.689}
Poor / Rich	1.558 [1.547]	1.657 [1.618]	1.462 [1.530]
4. Output, Constant Dollar GDP ($Y_{i,t}$)			
Poor	300.155 [35.945] {2.607}	149.502 [23.590] {2.107}	431.976 [43.736] {2.799}
Rich	833.906 [163.699] {2.279}	575.752 [106.539] {2.241}	1059.790 [239.000] {2.300}
Poor / Rich	0.360 [0.220]	0.260 [0.221]	0.407 [0.183]
5. Capital Stock, Private, Constant Dollars ($K_{i,t}^{P}$)			
Poor	322.215 [41.257] {2.212}	167.665 [28.104] {2.281}	457.446 [51.971] {2.324}
Rich	1395.628 [314.404] {2.001}	1017.569 [222.053] {1.945}	1726.431 [403.490] {2.040}
Poor / Rich	0.231 [0.131]	0.164 [0.127]	0.265 [0.129]
6. Relative Price Of Output To The Price Of Private Capital ($P_{i,t}^{Y} / P_{i,t}^{K^P}$)			
Poor	0.799 [0.809] {0.208}	0.642 [0.657] {0.363}	0.937 [0.942] {0.170}
Rich	1.002 [0.990] {0.156}	0.910 [0.959] {0.207}	1.081 [1.042] {0.158}
Poor / Rich	0.798 [0.816]	0.705 [0.686]	0.866 [0.904]

-- continued --

Table 1: Descriptive Statistics For Select Variables (continued)

	1970-2014	1970-1990	1991-2014
	(1)	(2)	(3)
7. Price Of Output ($P^{Y}_{i,t}$)			
Poor	0.334 [0.325] {0.283}	0.264 [0.248] {0.444}	0.395 [0.370] {0.267}
Rich	0.656 [0.669] {0.238}	0.417 [0.431] {0.245}	0.864 [0.877] {0.247}
Poor / Rich	0.509 [0.487]	0.632 [0.575]	0.457 [0.422]
8. Price Of Private Capital ($P^{K^P}_{i,t}$)			
Poor	0.485 [0.431] {0.516}	0.526 [0.379] {0.836}	0.449 [0.422] {0.307}
Rich	0.662 [0.661] {0.254}	0.469 [0.465] {0.288}	0.831 [0.857] {0.274}
Poor / Rich	0.732 [0.652]	1.120 [0.814]	0.540 [0.492]
9. Share Of Economic Activity In The Shadow Economy ($SE_{i,t}$)			
Poor	0.378 [0.380] {0.305}	0.383 [0.384] {0.304}	0.373 [0.370] {0.307}
Rich	0.190 [0.172] {0.446}	0.193 [0.177] {0.445}	0.187 [0.168] {0.447}
Poor / Rich	1.988 [2.203]	1.986 [2.167]	1.990 [2.200]
10. Ratio Of Capital Stocks, Government Relative To Private ($RK^{G,P}_{i,t}$)			
Poor	1.278 [0.547] {3.000}	1.776 [0.498] {4.188}	0.842 [0.576] {1.060}
Rich	0.557 [0.353] {1.130}	0.599 [0.359] {1.256}	0.521 [0.342] {1.046}
Poor / Rich	2.292 [1.550]	2.964 [1.385]	1.615 [1.684]

Notes: A glossary and details concerning variable definitions and sources can be found in Appendix B. The figures are the mean [median] {coefficient of variation = standard deviation / absolute value of the mean}. For a given $X_{i,t}$, we first compute the time mean ($\bar{X}_i$), and then compute the mean, median, and standard deviation reported in Table 1 based on $\bar{X}_i$.

Table 2: Spot MPK's. Background Moments And Ratios Of Moments Uniformly-Weighted Means

Category	1970-2014	1970-1990	1991-2014
	(1)	(2)	(3)
1. MPK[1] (Lucas)			
Poor	0.296 [0.236] {0.793}	0.294 [0.239] {0.961}	0.298 [0.239] {0.668}
Rich	0.212 [0.170] {0.763}	0.203 [0.145] {1.033}	0.219 [0.184] {0.572}
Poor / Rich	1.402 [1.386]	1.448 [1.652]	1.364 [1.298]
2. MPK[2] (Caselli and Feyrer)			
Poor	0.094 [0.095] {0.492}	0.068 [0.065] {0.598}	0.117 [0.121] {0.500}
Rich	0.157 [0.141] {0.312}	0.132 [0.118] {0.404}	0.180 [0.163] {0.326}
Poor / Rich	0.597 [0.673]	0.514 [0.549]	0.650 [0.741]
3. MPK[3] (Monge-Naranjo, Sánchez, and Santaeulàlia-Llopis)			
Poor	0.186 [0.166] {0.530}	0.143 [0.128] {0.778}	0.224 [0.208] {0.470}
Rich	0.180 [0.157] {0.457}	0.148 [0.126] {0.638}	0.208 [0.189] {0.396}
Poor / Rich	1.035 [1.058]	0.962 [1.013]	1.081 [1.100]

Notes: See Section I for a discussion of these three spot MPK's and equations (4), (5), and (6) for the definitions of MPK[1], MPK[2], and MPK[3]. A glossary and details concerning variable definitions and sources can be found in Appendix B. The figures are the uniformly-weighted mean, [median], and {coefficient of variation = standard deviation / absolute value of the uniformly-weighted mean}. The rows for Poor / Rich contain the ratios of means or medians.

Table 3: Spot MPK's Moments And Ratios Of Moments Three New Adjustments

Category	1970-2014	1970-1990	1991-2014
	(1)	(2)	(3)
1. MPK[1] (Lucas)			
Poor	0.241 [0.236] {0.793}	0.229 [0.239] {0.961}	0.246 [0.239] {0.668}
Rich	0.157 [0.170] {0.763}	0.138 [0.145] {1.033}	0.170 [0.184] {0.572}
Poor / Rich	1.541 [1.386]	1.655 [1.652]	1.447 [1.298]
2. MPK[2] (Caselli and Feyrer)			
Poor	0.110 [0.095] {0.492}	0.088 [0.065] {0.598}	0.131 [0.121] {0.500}
Rich	0.139 [0.141] {0.312}	0.120 [0.118] {0.404}	0.154 [0.163] {0.326}
Poor / Rich	0.788 [0.673]	0.734 [0.549]	0.848 [0.741]
3. MPK[3] (Monge-Naranjo, Sánchez, and Santaeulàlia-Llopis)			
Poor	0.185 [0.166] {0.530}	0.150 [0.128] {0.778}	0.219 [0.208] {0.470}
Rich	0.151 [0.157] {0.457}	0.129 [0.126] {0.638}	0.169 [0.189] {0.396}
Poor / Rich	1.222 [1.058]	1.162 [1.013]	1.297 [1.100]

--continued--

Table 3: Spot MPK's. New Data (cont.) Moments And Ratios Of Moments Three New Adjustments

Category	1970-2014	1970-1990	1991-2014
	(1)	(2)	(3)
4. MPK^4 (Shadow Economy Adjustment)			
Poor	0.241 [0.222] {0.543}	0.202 [0.176] {0.772}	0.283 [0.273] {0.494}
Rich	0.173 [0.187] {0.483}	0.147 [0.155] {0.659}	0.193 [0.219] {0.424}
Poor / Rich	1.399 [1.186]	1.369 [1.136]	1.469 [1.250]
5. MPK^5 (Government Capital Adjustment)			
Poor	0.173 [0.152] {0.534}	0.142 [0.120] {0.773}	0.204 [0.197] {0.477}
Rich	0.119 [0.123] {0.412}	0.101 [0.102] {0.530}	0.134 [0.143] {0.390}
Poor / Rich	1.449 [1.238]	1.404 [1.175]	1.523 [1.377]
6. MPK^6 (Shadow Economy And Government Capital Adjustments)			
Poor	0.226 [0.206] {0.548}	0.191 [0.166] {0.769}	0.264 [0.256] {0.502}
Rich	0.136 [0.142] {0.438}	0.116 [0.121] {0.555}	0.153 [0.163] {0.414}
Poor / Rich	1.659 [1.447]	1.652 [1.375]	1.725 [1.571]

Notes: See Section II for a discussion of these six spot MPK's, and equations (4) to (9) for the definitions of MPK^1 to MPK^6. A glossary and details concerning variable definitions and sources can be found in Appendix B. The figures are the capital-weighted mean and the uniformly-weighted [median], and {coefficient of variation = standard deviation / absolute value of the uniformly-weighted mean}. The rows for Poor / Rich contain the ratios of means or medians. For a given time interval, the capital weight is the time-averaged private capital stock for country *i* divided by the time-averaged private capital stock for all countries.

Table 4: Steady-State MPK's. Baseline Results
Moments, Ratios Of Moments, & Adjustment Parameters

Category	1970-2014	1970-1990	1991-2014
	(1)	(2)	(3)
1. MPK^6 (Shadow Economy And Government Capital Adjustments)			
Steady-State MPK's			
Poor	0.235 [0.220] {0.495}	0.180 [0.145] {0.793}	0.304 [0.299] {0.476}
Rich	0.144 [0.154] {0.411}	0.121 [0.134] {0.481}	0.172 [0.196] {0.421}
Poor / Rich	1.633 [1.428]	1.483 [1.089]	1.770 [1.523]
Adjustment Parameters			
λ^{Poor}	0.136 (0.039) /4.7/	0.276 (0.038) /2.1/	0.179 (0.093) /3.5/
λ^{Rich}	0.124 (0.029) /5.2/	0.112 (0.052) /5.8/	0.100 (0.052) /6.6/
2. MPK^5 (Government Capital Adjustment)			
Steady-State MPK's			
Poor	0.180 [0.164] {0.475}	0.133 [0.108] {0.796}	0.236 [0.223] {0.451}
Rich	0.126 [0.133] {0.380}	0.106 [0.112] {0.447}	0.150 [0.173] {0.390}
Poor / Rich	1.428 [1.227]	1.259 [0.966]	1.572 [1.288]
Adjustment Parameters			
λ^{Poor}	0.135 (0.039) /4.8/	0.276 (0.038) /2.1/	0.179 (0.093) /3.5/
λ^{Rich}	0.125 (0.029) /5.2/	0.112 (0.052) /5.8/	0.102 (0.053) /6.4/
3. MPK^4 (Shadow Economy Adjustment)			
Steady-State MPK's			
Poor	0.251 [0.229] {0.487}	0.190 [0.159] {0.795}	0.326 [0.326] {0.468}
Rich	0.182 [0.196] {0.411}	0.153 [0.165] {0.482}	0.220 [0.269] {0.418}
Poor / Rich	1.380 [1.171]	1.241 [0.963]	1.478 [1.210]
Adjustment Parameters			
λ^{Poor}	0.135 (0.039) /4.8/	0.277 (0.038) /2.1/	0.178 (0.093) /3.5/
λ^{Rich}	0.119 (0.028) /5.5/	0.112 (0.052) /5.8/	0.087 (0.050) /7.6/
4. MPK^3 (Baseline)			
Steady-State MPK's			
Poor	0.192 [0.171] {0.469}	0.141 [0.119] {0.799}	0.253 [0.245] {0.443}
Rich	0.159 [0.166] { 0.377}	0.134 [0.140] {0.447}	0.193 [0.222] {0.383}
Poor / Rich	1.206 [1.035]	1.053 [0.852]	1.315 [1.102]
Adjustment Parameters			
λ^{Poor}	0.135 (0.039) /4.8/	0.277 (0.038) /2.1/	0.177 (0.092) /3.6/
λ^{Rich}	0.120 (0.029) /5.4/	0.112 (0.052) /5.8/	0.089 (0.051) /7.4/

Notes: Table notes can be found on the next page.

Notes to Table 4: See Section IV for a discussion of these four steady-state MPK's, equations (9), (8), (7), and (6) for the definitions of MPK^6, MPK^5, MPK^4, and MPK^3, and equation (15) for the mapping from estimated parameters (determined by the spot MPK's) into steady-state MPK's. A glossary and details concerning variable definitions and sources can be found in Appendix B. The figures are the capital-weighted mean and the uniformly-weighted [median], {coefficient of variation = standard deviation / absolute value of the uniformly-weighted mean}, (standard errors), and /half-lives in years/. For a given time interval, the capital weight is the time-averaged private capital stock for country i divided by the time-averaged private capital stock for all countries. The Poor and Rich MPK's and the Poor/Rich ratio are estimated precisely; these statistics are at least nine times larger than their corresponding standard errors. The λ's are bias adjusted per the discussion in Section IV.D, $\hat{\lambda} = \left(\tilde{\lambda}(T-1)-2\right)/\left(T-2\right)$, where $\tilde{\lambda}$ is the initial estimate and T is the time dimension of the regression. Standard errors for $\hat{\lambda}$ are computed by the delta method. Half-lives are computed by the following equation: $Ln(2)/\ln(1-\hat{\lambda})$.

Table 5: Steady-State MPK[6]. Robustness Checks
Moments, Ratios Of Moments, & Adjustment Parameters

Category	**1970-2014**	**1970-1990**	**1991-2014**
	(1)	(2)	(3)
1. Two Lags Of The Dependent Variable			
Steady-State MPK's			
Poor	0.244 [0.233] {0.474}	0.180 [0.145] {0.798}	0.301 [0.294] {0.474}
Rich	0.143 [0.154] {0.404}	0.120 [0.131] {0.469}	0.160 [0.177] {0.403}
Poor / Rich	1.704 [1.517]	1.496 [1.107]	1.885 [1.663]
Adjustment Parameters			
λ^{Poor}	0.112 (0.034)	0.279 (0.048)	0.169 (0.088)
λ^{Rich}	0.120 (0.028)	0.102 (0.045)	0.119 (0.052)
2. Equal Adjustment Parameters			
Steady-State MPK's			
Poor	0.235 [0.221] {0.494}	0.178 [0.146] {0.795}	0.309 [0.308] {0.475}
Rich	0.143 [0.153] {0.412}	0.117 [0.125] {0.492}	0.163 [0.180] {0.413}
Poor / Rich	1.646 [1.444]	1.515 [1.168]	1.898 [1.714]
Adjustment Parameters			
$\lambda^{Poor} = \lambda^{Rich}$	0.134 (0.036)	0.233 (0.0301)	0.165 (0.085)
3. Alternative Definitions Of Rich And Poor Countries			
Steady-State MPK's			
Poor	0.239 [0.219] {0.555}	0.158 [0.144] {0.931}	0.327 [0.304] {0.504}
Rich	0.154 [0.160] {0.436}	0.125 [0.126] {0.480}	0.192 [0.227] {0.481}
Poor / Rich	1.556 [1.369]	1.265 [1.149]	1.704 [1.341]
Adjustment Parameters			
λ^{Poor}	0.136 (0.047)	0.240 (0.040)	0.188 (0.102)
λ^{Rich}	0.130 (0.024)	0.223 (0.045)	0.108 (0.045)

Notes: See Section V for a discussion of these robustness checks and the notes to Table 4 for further details about the table entries. All results in Table 5 should be compared to the baseline results for MPK[6] in panel 1 of Table 4. The variations from the baseline model are as follows: panel 1, the dependent variable lagged two periods as an additional regressor (λ^{Poor} and λ^{Rich} are computed as the sum of the two coefficients on the lagged dependent variables); panel 2, $\lambda^{Poor} = \lambda^{Rich}$; panel 3, the sets of countries defining the poor and rich groups are defined in terms of the four-way World Bank country classification (by income) as follows: Poor' = {Low, Lower Middle}, Rich' = {Upper Middle, High}. See the Notes to Table 4 for additional details.

Table 6: Misallocation Of Private Capital
Percentage Changes (Absolute Values) In The Private Capital Stock

Change In MPK's	1970 2014	1970 1990	1991 2014	1970-2014		1970-1990		1991-2014	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	------- Global	-------		Poor	Rich	Poor	Rich	Poor	Rich
A. $\overline{\text{MPK}}_i^{\text{Spot}} - \text{MPK}^{**}$									
$\sigma = 0.5$	17.7	14.0	21.1	23.6	15.3	28.7	9.8	20.7	21.4
$\sigma = 1.0$	35.4	28.0	42.3	47.2	30.6	57.4	19.5	41.3	42.7
$\sigma = 1.5$	53.1	42.0	63.4	70.9	45.9	86.1	29.3	62.0	64.1
B. $\overline{\text{MPK}}_i^{\text{Spot}} - \text{MPK}_i^{*}$									
$\sigma = 0.5$	3.5	5.0	8.9	3.4	3.6	4.8	5.1	10.1	8.3
$\sigma = 1.0$	7.1	10.0	17.7	6.8	7.2	9.5	10.1	20.3	16.5
$\sigma = 1.5$	10.6	15.0	26.6	10.2	10.8	14.3	15.2	30.4	24.8
C. $\text{MPK}_i^{*} - \text{MPK}^{**}$									
$\sigma = 0.5$	16.2	16.0	20.2	24.0	13.1	27.1	12.8	24.0	18.4
$\sigma = 1.0$	32.4	31.9	40.3	48.0	26.1	54.2	25.5	47.9	36.8
$\sigma = 1.5$	48.6	47.9	60.5	72.0	39.2	81.3	38.3	71.9	55.2

Notes: See Section VII for a discussion of these results and Section VIII and the notes to Table 7 for a discussion of the construction of MPK**. All entries are the absolute values of the difference between two MPK's. The global entries in columns 1, 2, and 3 can be interpreted as approximate weighted averages of the corresponding entries for poor and rich countries in columns 4 & 5, 6 & 7, and 8 & 9, with weights approximately equal to 0.30 for poor countries and 0.70 for rich countries. In a given panel, the decomposition of the change in the second and third rows does not equal the total change because different bases are used to compute the percentage changes.

Table 7: Reallocation Of Private Capital. Equalizing MPK's Subject To A Capital Constraint Percentage Changes In Output

Adjusting MPK's	1970 2014	1970 1990	1991 2014	1970-2014		1970-1990		1991-2014	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	------- Global -------			Poor	Rich	Poor	Rich	Poor	Rich
A. Maximize Global Output				$(MPK^{**} = 0.156)$		$(MPK^{**} = 0.124)$		$(MPK^{**} = 0.187)$	
$\sigma = 0.5$	1.2	1.0	1.5	4.9	-1.1	4.2	-0.4	5.7	-1.6
$\sigma = 1.0$	2.4	2.1	2.9	9.9	-2.2	8.5	-0.9	11.4	-3.1
$\sigma = 1.5$	3.7	3.1	4.4	14.8	-3.4	12.7	-1.3	17.1	-4.7
B. Maximize Global Output With No Loss to Rich				$(MPK^{**} = 0.156)$		$(MPK^{**} = 0.124)$		$(MPK^{**} = 0.187)$	
$\sigma = 0.5$	1.2	1.0	1.5	3.2	0.0	3.3	0.0	3.5	0.0
$\sigma = 1.0$	2.4	2.1	2.9	6.3	0.0	6.6	0.0	7.1	0.0
$\sigma = 1.5$	3.7	3.1	4.4	9.5	0.0	9.9	0.0	10.6	0.0
C. Maximize Global Output (MPK^3)				$(MPK^{**} = 0.163)$		$(MPK^{**} = 0.127)$		$(MPK^{**} = 0.200)$	
$\sigma = 0.5$	0.8	0.6	1.0	2.6	-0.3	1.0	0.4	3.4	-0.6
$\sigma = 1.0$	1.6	1.1	2.1	5.1	-0.5	2.0	0.7	6.7	-1.3
$\sigma = 1.5$	2.5	1.7	3.1	7.7	-0.8	2.9	1.1	10.1	-1.9

Notes: See Section VIII for a discussion of these results. MPK** is chosen as follows: Panel A, to maximize global output by equating MPK's across all countries subject to a capital constraint; Panel B, in a manner similar to that in Panel A with the additional constraint that output levels in rich countries as a group do not change by a transfer of output from poor to rich countries; Panel C, in a manner similar to that in Panel A with MPK^3 used in place of MPK^6. The MPK**'s vary across the three intervals and between Panels A, B, and C and are presented in columns (4)/(5), (6)/(7), and (8)/(9). The percentage changes in output reported in this table are based on the percentage changes in MPK^{**} relative to country-specific steady-state MPK's (MPK_i^*'s).

International Capital Allocations

And

The Lucas Paradox Redux

Robert S. Chirinko

And

Debdulal Mallick*

August 2019

Appendices

Appendix A: List Of 88 Countries

Table A.1

Country	Country Code	Income Group (4 Categories; World Bank)	Rich/Poor (2 Categories)
(1)	(2)	(3)	(4)
A. 32 Rich Countries (32 High Income Countries)			
Australia	AUS	High	Rich
Austria	AUT	High	Rich
Bahamas	BHS	High	Rich
Bahrain	BHR	High	Rich
Belgium	BEL	High	Rich
Canada	CAN	High	Rich
Chile	CHL	High	Rich
China, Hong Kong SAR	HKG	High	Rich
Cyprus	CYP	High	Rich
Finland	FIN	High	Rich
France	FRA	High	Rich
Germany	DEU	High	Rich
Iceland	ISL	High	Rich
Ireland	IRL	High	Rich
Israel	ISR	High	Rich
Italy	ITA	High	Rich
Japan	JPN	High	Rich
Kuwait	KWT	High	Rich
Luxembourg	LUX	High	Rich
Malta	MLT	High	Rich
Netherlands	NLD	High	Rich
New Zealand	NZL	High	Rich
Norway	NOR	High	Rich
Oman	OMN	High	Rich
Republic of Korea	KOR	High	Rich
Singapore	SGP	High	Rich
Sweden	SWE	High	Rich
Switzerland	CHE	High	Rich
Trinidad and Tobago	TTO	High	Rich
United Kingdom	GBR	High	Rich
United States	USA	High	Rich
Uruguay	URY	High	Rich
-- continued --			

Table A.1 (continued)

Country	Country Code	Income Group (4 Categories; World Bank)	Rich/Poor (2 Categories)
(1)	(2)	(3)	(4)
B. 56 Poor Countries (23 Upper Middle Income Countries)			
Argentina	ARG	Upper Middle	Poor
Botswana	BWA	Upper Middle	Poor
Brazil	BRA	Upper Middle	Poor
China	CHN	Upper Middle	Poor
Colombia	COL	Upper Middle	Poor
Costa Rica	CRI	Upper Middle	Poor
Dominican Republic	DOM	Upper Middle	Poor
Ecuador	ECU	Upper Middle	Poor
Fiji	FJI	Upper Middle	Poor
Gabon	GAB	Upper Middle	Poor
Iran (Islamic Republic of)	IRN	Upper Middle	Poor
Lebanon	LBN	Upper Middle	Poor
Malaysia	MYS	Upper Middle	Poor
Mauritius	MUS	Upper Middle	Poor
Mexico	MEX	Upper Middle	Poor
Namibia	NAM	Upper Middle	Poor
Panama	PAN	Upper Middle	Poor
Paraguay	PRY	Upper Middle	Poor
Peru	PER	Upper Middle	Poor
South Africa	ZAF	Upper Middle	Poor
Suriname	SUR	Upper Middle	Poor
Thailand	THA	Upper Middle	Poor
Venezuela	VEN	Upper Middle	Poor

-- continued --

Table A.1 (continued)

Country	Country Code	Income Group (4 Categories; World Bank)	Rich/Poor (2 Categories)
(1)	(2)	(3)	(4)
	B. 56 Poor Countries (22 Lower Middle Income Countries)		
Bolivia	BOL	Lower Middle	Poor
Cameroon	CMR	Lower Middle	Poor
Côte d'Ivoire	CIV	Lower Middle	Poor
Egypt	EGY	Lower Middle	Poor
Guatemala	GTM	Lower Middle	Poor
Honduras	HND	Lower Middle	Poor
India	IND	Lower Middle	Poor
Indonesia	IDN	Lower Middle	Poor
Jordan	JOR	Lower Middle	Poor
Kenya	KEN	Lower Middle	Poor
Lao People's DR	LAO	Lower Middle	Poor
Lesotho	LSO	Lower Middle	Poor
Mauritania	MRT	Lower Middle	Poor
Mongolia	MNG	Lower Middle	Poor
Morocco	MAR	Lower Middle	Poor
Nicaragua	NIC	Lower Middle	Poor
Nigeria	NGA	Lower Middle	Poor
Philippines	PHL	Lower Middle	Poor
Sri Lanka	LKA	Lower Middle	Poor
Sudan (Former)	SDN	Lower Middle	Poor
Swaziland	SWZ	Lower Middle	Poor
Tunisia	TUN	Lower Middle	Poor

-- continued --

Table A.1 (continued)

Country	Country Code	Income Group (4 Categories; World Bank)	Rich/Poor (2 Categories)
(1)	(2)	(3)	(4)
B. 56 Poor Countries (11 Low Income Countries)			
Benin	BEN	Low	Poor
Burkina Faso	BFA	Low	Poor
Central African Republic	CAF	Low	Poor
Chad	TCD	Low	Poor
Guinea	GIN	Low	Poor
Mozambique	MOZ	Low	Poor
Niger	NER	Low	Poor
Rwanda	RWA	Low	Poor
Senegal	SEN	Low	Poor
Sierra Leone	SLE	Low	Poor
U.R. Tanzania: Mainland	TZA	Low	Poor

Notes: Countries are excluded in the sample of 88 countries used in this paper if they (i) have produced capital stock (K^{PRO}), labor income share (LIS), or output (Y) data missing during any part of the 1970-2014 sample period; (ii) are ex-communist countries; or (iii) have negative values for our preferred measure of MPK (MPK^6) during any part of the sample period. Zimbabwe is excluded because of its extremely high values for MPK^6. The Rich/Poor groups are defined in terms of the four income groups taken from the World Bank classification: Rich ={High}, Poor = {Upper Middle, Lower Middle, Low}. The results in Table 5, panel 3 are based on an alternative definition of the Rich/Poor groups: Rich' ={High, Upper Middle}, Poor' = {Lower Middle, Low}.

Appendix B: Glossary: Variable Definitions And Sources

Notes:

1. Latin letters define variables. Greek letters define econometric parameters and calibrated parameters or variables.

2. International prices are purchasing power parity prices. “A country’s PPP gives the number of local currency units (e.g. euro’s) that are needed to buy a bundle of products worth one dollar in the US. Dividing the PPP by the nominal exchange rate (also in local currency units per dollar) then gives the “price level” of that country relative to the US. A price level of 0.5, for example, indicates that local prices converted to US dollars with the nominal exchange rate are ½ as high on average as in the United States, as might be the case for a developing country” (PWT-USER, p. 2, fn. 2).

3. Some of the text below is taken directly from government documents that are in the public domain.

Roman Letters

$CIS^{N}_{i,t}$ Capital income share for natural capital. Source: WB-WDI (NY.GDP.TOTL.TR.ZS).Capital income for natural capital is the sum of rents from oil, natural gas, coal (hard and soft), minerals, and forests. For many countries, especially the developing countries, the rent share data are not available for the entire 1970-2014 period. However, data are usually missing at the tails. The missing data for beginning periods are replaced by the value for the earliest available period, and the missing data for end periods are replaced by the value for the most recently available period.

$CIS^{P}_{i,t}$ Capital income share for private capital based on an adjustment using flow data. Transformation: $\left(CIS^{P+G+N}_{i,t} - CIS^{N}_{i,t}\right) / \left(1 + \phi_g * RK^{G,P}_{i,t}\right)$. Note if $\phi_g = 1$, this transformation reduces to a simple weighted average, $\left(CIS^{P+G+N}_{i,t} - CIS^{N}_{i,t}\right) * \left(K^{P}_{i,t} / (K^{P}_{i,t} + K^{G}_{i,t})\right)$.

$CIS^{P+G}_{i,t}$ Capital income share for private and government (public) capital (also referred to as produced capital) based on an adjustment using flow data. Transformation: $CIS^{P+G+N}_{i,t} - CIS^{N}_{i,t}$.

$CIS^{P+G+N}_{i,t}$ Capital income share for private, government, and natural capital. Transformation: $1 - LIS_{i,t}$.

F[.] Neoclassical production function.

g Subscript indexing a group of countries. Two groupings are employed in this study based on the four-way World Bank country classification (by income) defined by WB-CLG: high, upper middle, lower middle, low. For the 2018 fiscal year, high / upper middle / lower middle / low economies are defined as having Gross National Income (formerly Gross National Product) per capita falling in the following intervals: greater than $12,236 / $12,235 to $3,956 / $3,955 to $1,006 / less than $1,005. These figures are calculated with 2016 data using the method in WB-ATLAS. See Appendix A for a list of the countries included in each of the groupings.

The first grouping defines the rich group as the high income countries and the poor group as the sum of the upper middle, lower middle, and low income countries.

The second grouping defines the rich group as the sum of the high income and upper middle countries and the poor group as the sum of the lower middle and low income countries.

i Subscript indexing a country. See Appendix A for a list of the countries, organized by groups. See the entry above for "g" for further information about the groups.

ICA_t^D Intellectual capital adjustment, depreciation rate. Source: BEA-FAA. This variable is constructed on a current-cost basis as the difference between the depreciation rate for equipment, structures, and intellectual property capital less the depreciation rate for only equipment and structures capital. The depreciation rate for equipment, structures, and intellectual property capital is the ratio of the depreciation flow for these three components (the sum of the entries in Table 2.4, rows 9, 42, and 83) and the net stock of these three components (the sum of the entries in Table 2.1, rows 9, 42, and 83). The depreciation rate for equipment and structures capital is defined analogously; the ratio of the sum of the entries in Table 2.4, rows 9 and 42 and the sum of the entries in Table 2.1, rows 9 and 42. The equipment and structures figures include both non-residential and residential capital.

ICA_t^K Intellectual capital adjustment, net capital stock. Source: BEA-FAA. This variable is constructed on a current-cost basis as the ratio of the net capital stock for intellectual property capital (entry in Table 2.1, row 83) and the net capital stock for equipment and structures capital (the sum of the entries in Table 2.1, rows 9 and 42). The equipment and structures figures include both non-residential and residential capital.

j Subscript indexing an industry. This index appears only in the theoretical model.

$K^{G}_{i,t}$ Government (public) capital stock, billions of constant 2011 international dollars. Source: IMF (kgov_rppp).

$K^{N@}_{i,t}$ Natural capital stock, constant 2014 U.S. dollars.

Source: WB-WN-2018 (NW.NCA.TO). The @ superscript indicates that these data are not strictly comparable to the other capital stock data. Natural capital includes the valuation of fossil fuel energy (oil, gas, hard and soft coal) and minerals (bauxite, copper, gold, iron ore, lead, nickel, phosphate, silver, tin, and zinc), agricultural land (cropland and pastureland), forests (timber and some nontimber forest products), and protected areas. Values are measured at market exchange rates in constant 2014 US dollars, using a country-specific GDP deflator. Data are available for 1995, 2000, 2005, 2010 and 2014. Values for the pre-1995 period have been extrapolated with the 1995 value. Values for the 1996-1999 period have been interpolated by the mean of the 1995 and 2000 values. Values for 2001-2004 have been interpolated by the mean of the 2000 and 2005 values. Values for 2006-2009 have been interpolated by the mean of the 2005 and 2010 values. Values for 2011-2013 have been interpolated by the mean of 2010 and 2014.

$K^{P}_{i,t}$ Private capital stock, billions of constant 2011 international dollars.

Source: IMF (kpriv_rppp).

$K^{P+G}_{i,t}$ Private plus government (public) capital stock, billions of constant 2011 international dollars. Also referred to as the produced capital stock. Transformation: $K^{P}_{i,t} + K^{G}_{i,t}$.

$K^{P@+G@}_{i,t}$ Produced capital, constant 2014 US$. Source: WB-WN-2018 (NW.PCA.TO).

The @ superscript indicates that these data are not strictly comparable to the other capital stock data. Produced capital includes the value of machinery, buildings, equipment, and residential and nonresidential urban land. Values are measured at market exchange rates in constant 2014 US dollars, using a country-specific GDP deflator. Data are available for 1995, 2000, 2005, 2010 and 2014. The imputations are the same as for $K^{N@}_{i,t}$.

$L_{i,t}$ Labor input.

$LIS_{i,t}$ Labor income share, ratio of labor compensation to GDP all in current national prices. Source: PWT-9 (labsh).

$MPK^{1}_{i,t}$ [Lucas] Marginal product of capital. Transformation:

$CIS^{P+G+N}_{i,t} * \left(Y_{i,t} / K^{P+G}_{i,t} \right)$.

$MPK^{2}_{i,t}$ [Caselli and Feyrer, CF] Marginal product of capital with the CF corrections for natural capital and relative prices. Transformation:

$CIS^{P+G+N}_{i,t} * \left(Y_{i,t} / K^{P+G}_{i,t} \right) * \left(P^{Y}_{i,t} / P^{K^{P+G}}_{i,t} \right) * \left(K^{P@+G@}_{i,t} / (K^{N@}_{i,t} + K^{P@+G@}_{i,t}) \right)$.

Due to restrictions on data availability, the capital stock data used in the first term in parentheses is from the IMF, while the capital stock data used in the third term in parentheses is from WB-WN-2018. (In their original article, CF used data from WB-WN-2006, which was the most currently available data at that time.) In the ideal but unattainable situation where all the data would be from the same source and price deflators would be available for all types of capital, the above formula would reduce to the following,

$CIS^{P+G+N}_{i,t} * \left(Y_{i,t} / K^{P+G+N}_{i,t} \right) * \left(P^{Y}_{i,t} / P^{K^{P+G+N}}_{i,t} \right)$.

$MPK^{3}_{i,t}$ [Monge-Naranjo, Sánchez, and Santaeulàlia-Llopis, MSS] Marginal product of capital with the MSS correction for natural capital and the CF correction for relative prices. Transformation:

$\left(CIS^{P+G+N}_{i,t} - CIS^{N}_{i,t} \right) * \left(Y_{i,t} / K^{P+G}_{i,t} \right) * \left(P^{Y}_{i,t} / P^{K^{P+G}}_{i,t} \right)$.

$MPK^{4}_{i,t}$ [Chirinko and Mallick, CM: SE] Marginal product of capital with the MSS correction for natural capital, the CF correction for relative prices, and the CM correction for the shadow economy. Transformation:

$\left(CIS^{P+G+N}_{i,t} - CIS^{N}_{i,t} \right) * \left(Y_{i,t} / K^{P+G}_{i,t} \right) * \left(P^{Y}_{i,t} / P^{K^{P+G}}_{i,t} \right) * \left(1 + SE_{i,t} \right)$.

$MPK^{5}_{i,t}$ [Chirinko and Mallick, CM: GOV] Marginal product of capital with the MSS correction for natural capital, the CF correction for relative prices, and the CM correction for government capital, where $\phi_{rich} = 1.9$ and $\phi_{poor} = 1.2$.

Transformation:

$\left(CIS^{P+G+N}_{i,t} - CIS^{N}_{i,t} \right) * \left(Y_{i,t} / K^{P}_{i,t} \right) * \left(P^{Y}_{i,t} / P^{K^{P}}_{i,t} \right) / \left(1 + \phi_{g} * RK^{G,P}_{i,t} \right)$.

$MPK^{6}_{i,t}$ [Chirinko and Mallick, CM: SE+GOV] Marginal product of capital with the MSS correction for natural capital, the CF correction for relative prices, and the CM corrections for the shadow economy and for government capital, where $\phi_{rich} = 1.9$ and $\phi_{poor} = 1.2$. Transformation: $\left(CIS^{P+G+N}_{i,t} - CIS^{N}_{i,t}\right) * \left(Y_{i,t} / K^{P}_{i,t}\right) * \left(P^{Y}_{i,t} / P^{K^P}_{i,t}\right) * \left(1 + SE_{i,t}\right) / \left(1 + \phi_g * RK^{G,P}_{i,t}\right)$.

$P^{K^P}_{i,t}$ Price index for private capital, 2011 base year. Transformation: equal to $P^{K^{P+G}}_{i,t}$.

$P^{K^{P+G}}_{i,t}$ Price index for private plus government capital, 2011 base year. Source: PWT-9 (pl_k). This variable is measured as the price index for new capital goods (i.e., investment).

$P^{K^{P+G+N}}_{i,t}$ Price index for private plus government plus natural capital, 2011 base year. Transformation: equal to $P^{K^{P+G}}_{i,t}$.

$P^{Y}_{i,t}$ Price index for output, 2011 base year. Source: PWT-9 (pl_gdpo).

r Discount rate, $0 < r < 1$. This variable appears only in the theoretical model.

R Discount factor equal to $(1+r)^{-1} < 1$. This variable appears only in the theoretical model.

$RK^{G,P}_{i,t}$ Ratio of capital stocks, government relative to private. Transformation: $K^{G}_{i,t} / K^{P}_{i,t}$.

$SE_{i,t}$ Share of economic activity in the shadow economy, measured as a percentage of reported GDP. Source: SBM (Table 2, pp. 454-456). Definition: “The shadow economy includes all market-based legal production of goods and services that are deliberately concealed from public authorities for any of the following reasons: (1) to avoid payment of income, value added or other taxes, (2) to avoid payment of social security contributions, (3) to avoid having to meet certain legal labor market standards, such as minimum wages, maximum working hours, safety standards, etc., and (4) to avoid complying with certain administrative procedures, such as completing statistical questionnaires or other administrative forms” (p. 444). Data for 1970-1998 have been extrapolated by the 1999 value. Data for 2008-2014 have been extrapolated by the 2007 value.

t Subscript indexing calendar time. t = {1970, 2014}.

$wt_{i,j,t}$ A fixed weight representing the relative importance of industry j in country i at time t.

$Y_{i,t}$ Output, gross domestic product, billions of constant 2011 international dollars. Source: IMF (GDP_rppp).

Greek Letters

α_i Country fixed effect. Estimated econometrically.

δ_i^G Depreciation rate for government capital. Source: IMF-FAD. The depreciation rates are only available for high, middle, and low income countries as a group, and they are the averages of values for 1960 and 2015: Low = 0.0250, Middle = 0.0303, High = 0.0360.

δ_i^P Depreciation rate for private capital. Source: IMF-FAD. The depreciation rates are only available for high, middle, and low income countries as a group, and they are the averages of values for 1960 and 2015: Low = 0.0425, Middle = 0.0628, High = 0.0751.

$\delta_{i,t}^{P+G}$ Depreciation rate for private and government (produced) capital. Transformation: $\delta_i^P\,(K_{i,t-1}^P / K_{i,t-1}^{P+G}) + \delta_i^G\,(K_{i,t-1}^G / K_{i,t-1}^{P+G})$.

$\varepsilon_{i,g,t}$ White-noise error term. Estimated econometrically.

ζ^A Adjustment cost parameter that translates changes in MPK's into pecuniary costs. This parameter appears only in the theoretical model.

ζ^M Misallocation cost parameter that translates deviations of MPK's from MPK* into pecuniary costs. This parameter appears only in the theoretical model.

λ_g Adjustment parameter. Estimated econometrically.

$\tau_{g,t}$ Time fixed effect for group g. Estimated econometrically.

$\upsilon_{i,g,t}$ Composite error term, equal to $\varepsilon_{i,g,t} + \tau_{g,t}$. Transformation based on the two components.

φ_g Ratio of government MPK's to private MPK's for group g. Source: Computed from LLP (2012, Table 1). Data are available only as a cross-section. For a given country, we compute the ratio of the marginal product of government capital to the marginal product of private capital. These country-specific ratios are then sorted into poor and rich groups, and averaged: $\varphi_{poor} = 1.2$ and $\varphi_{rich} = 1.9$. These estimates reflect two economic forces. The general phenomenon of the underprovision of government goods is consistent with both ratios being greater than one. The greater proportions of government to private capital in poor countries relative to rich countries (cf. $RK^{G,P}_{i,t}$ in Table 1, panel 10 and the discussion in Section II) is consistent with $\varphi_{poor} < \varphi_{rich}$.

Legend

BEA- FAA: Bureau of Economic Analysis, *National Data, Fixed Assets Accounts Tables, Section 2 Private Fixed Assets By Type. Table 2.1. Current-Cost Net Stock of Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type; Table 2.4. Current-Cost Depreciation of Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type.* https://apps.bea.gov/iTable/iTable.cfm?ReqID=10&step=2 .

IMF: International Monetary Fund, *The IMF and Public Investment Management.* http://www.imf.org/external/np/fad/publicinvestment/ .

IMF-FAD: International Monetary Fund, *FAD Investment And Capital Stock Database 2017: Manual & Faq - Estimating Public, Private, And PPP Capital Stocks*, p. 4.

LLP: Lowe, Matt, Papageorgiou, Chris, and Perez-Sebastian, Fidel, "The Public and Private MPK," International Monetary Fund (July 2012).

PWT-9: Groningen Growth and Development Centre. *Penn World Table, Version 9*. www.ggdc.net/pwt .

PWT-USER: Feenstra, Robert C., Inklaar, Robert, and Timmer, Marcel, "PWT 8.0 – a user guide." https://irs.princeton.edu/sites/irs/files/event/uploads/PWT%2080%20-%20a%20user%20guide.pdf .

SBM: Schneider, Friedrich , Buehn, Andreas, and Montenegro, Claudio E.,"New Estimates for the Shadow Economies all over the World," *International Economic Journal* 24/4 (December 2010), 443-461. DOI: 10.1080/10168737.2010.525974.

WB-ATLAS: World Bank, *The World Bank Atlas method – detailed methodology*. https://datahelpdesk.worldbank.org/knowledgebase/articles/378832-what-is-the-world-bank-atlas-method .

WB-CLG: World Bank, Country and Lending Groups, *Data*. https://datahelpdesk.worldbank.org/knowledgebase/articles/906519-world-bank-country-and-lending-groups .

WB-WDI: World Bank, *World Development Indicators*. https://databank.worldbank.org/data/source/world-development-indicators .

WB-WN-2006: World Bank, *Where is the Wealth of Nations? Measuring Capital for the 21st Century [2006]*. https://datacatalog.worldbank.org/dataset/wealth-accounting .

WB-WN-2018: World Bank, *The Changing Wealth of Nations 2018: Building a Sustainable Future.* https://datacatalog.worldbank.org/dataset/wealth-accounting .

Appendix C: Analysis Of MPK^3 And Data Sources

This appendix contains some additional results with the MPK^3 model developed by Monge-Naranjo, Sánchez, and Santaeulàlia-Llopis (2019, henceforth MSS) and estimated here with different datasets – the Penn World Table 8 (PWT8), PWT9, and the IMF – and with different sources for the capital income share for natural capital variable. To anticipate the results below, we find that the different datasets matter a great deal but that the results are robust to using different sources for the capital income share for natural capital variable.

We follow MSS' specification of the data series in PWT8 used to construct MPK^3. Results based on PWT8 are presented in column 1 of Table C.1. Similar to the results reported in MSS, we find evidence consistent with the LP; the Poor/Rich ratio is 1.32 and 1.21 in the earlier and later periods, where the later period ends in 2005 owing to the availability of MSS' data for the capital income share for natural capital. These results are also consistent with MSS' finding that the LP has become less severe in more recent years.

Column 2 of Table C.1 reports results based on PWT9. The Poor/Rich ratios are 1.26 in both the earlier and later periods. Thus, the LP is again supported; however, it does not fall in the later period.

The results change markedly with the IMF data. In column 3 of Table C.1, the Poor/Rich ratios are 0.93 and 1.02 for the early and later periods. These results indicate that the LP does not exist.

Since the constraint determining the end of the sample in 2005 is the capital income share for natural capital variable, we obtain an alternative data source from the World Bank (see the CIS^N variable in Appendix B for details). Before proceeding further with the World Bank data, we want to evaluate the extent to which it delivers results that differ from those using the MSS data. Table C.2 repeats the previous exercise with the only one change – substituting the World Bank data for the MSS data in defining the capital income share for natural capital variable. The results prove very robust for all three datasets (PWT8, PWT9, and IMF).

Figures 1 and 2 in the manuscript show the sharp rise in MPK's post 2005, and thus the inclusion of the nine additional years (2006 to 2014) is likely to be important in understanding MPK's.

Table C.1: Spot MPK^3's. Alternative Datasets Moments And Ratios Of Moments Uniformly-Weighted Means

	PWT8	PWT9	IMF
	(1)	(2)	(3)
	1. 1970-2005		
Poor	0.178 [0.163] {0.372}	0.178 [0.171] {0.410}	0.161 [0.165] {0.351}
Rich	0.139 [0.130] {0.327}	0.141 [0.124] {0.337}	0.166 [0.148] {0.554}
Poor/Rich	1.275 [1.257]	1.260 [1.378]	0.969 [1.114]
	2. 1970-1990		
Poor	0.185 [0.165] {0.397}	0.182 [0.167] {0.405}	0.145 [0.142] {0.415}
Rich	0.140 [0.124] {0.360}	0.144 [0.124] {0.383}	0.156 [0.133] {0.680}
Poor/Rich	1.322 [1.323]	1.260 [1.351]	0.929 [1.069]
	3. 1991-2005		
Poor	0.167 [0.154] {0.382}	0.172 [0.156] {0.458}	0.183 [0.184] {0.391}
Rich	0.138 [0.131] {0.318}	0.136 [0.125] {0.304}	0.180 [0.167] {0.458}
Poor/Rich	1.209 [1.176]	1.261 [1.251]	1.019 [1.100]

Notes: See Section I for a discussion of and equation (6) for the definition of MPK^3. A glossary and details concerning variable definitions and sources can be found in Appendix B. The figures are the uniformly-weighted mean, [median], and {coefficient of variation = standard deviation / absolute value of the uniformly-weighted mean}. The rows for Poor / Rich contain the ratios of means or medians. The sample was determined by the full sample of MSS firms and then placed into Poor and Rich groups based on the World Bank country classification (by income). There are 36 poor and 40 rich countries in columns 1 and 2; 35 poor and 40 rich countries in column 3. The later period ends in 2005 because of the availability of MSS' data for the capital income share for natural capital.

Table C.2: Spot MPK[3]'s. Alternative Source Of The Capital Income Share For Natural Capital Variable Moments And Ratios Of Moments Uniformly-Weighted Means

	PWT8	PWT9	IMF
	(1)	(2)	(3)
	1. 1970-2005		
Poor	0.182 [0.169] {0.358}	0.183 [0.175] {0.384}	0.167 [0.177] {0.318}
Rich	0.138 [0.129] {0.295}	0.140 [0.140] {0.303}	0.168 [0.145] {0.529}
Poor/Rich	1.321 [1.304]	1.309 [1.256]	0.993 [1.218]
	2. 1970-1990		
Poor	0.184 [0.164] {0.396}	0.181 [0.168] {0.398}	0.144 [0.139] {0.381}
Rich	0.135 [0.124] {0.308}	0.139 [0.123] {0.342}	0.153 [0.126] {0.623}
Poor/Rich	1.364 [1.331]	1.299 [1.360]	0.939 [1.099]
	3. 1991-2005		
Poor	0.180 [0.166] {0.352}	0.185 [0.167] {0.413}	0.199 [0.193] {0.362}
Rich	0.142 [0.130] {0.305}	0.140 [0.126] {0.282}	0.189 [0.170] {0.479}
Poor/Rich	1.265 [1.279]	1.322 [1.331]	1.054 [1.137]

Notes: See the notes to Table C.1. The only important difference is that MSS' capital income share for natural capital is replaced by a different measure constructed by the World Bank, labeled CIS^N. See Appendix B for details about the construction of CIS^N. The minor difference is that there is one fewer country in the rich group. There are 36 poor and 39 rich countries in columns 1 and 2; 35 poor and 39 rich countries in column 3.

Appendix D: Iterative Scheme For Computing The Global Reallocation Of Private Capital And The Increase In Global Output

This global optimum MPK is calculated according to the following four-step iterative scheme. First, we begin by assuming an arbitrary initial value for MPK^{**}, $\widehat{MPK_1}^{**}$, where the subscript 1 indicates the first iteration. Second, this initial estimate determines for the first iteration the percentage changes in the MPK's (from the country-specific steady-state MPK's, the MPK_i^*'s) that equate MPK's across all countries,

$$\text{(D-1)} \qquad \alpha_{i,1} = \frac{\widehat{MPK_1}^{**} - MPK_i^*}{MPK_i^*} = dMPK_i \,/\, MPK_i \,.$$

Third, a discrete version of equation (19) determines the private capital stock associated with $\alpha_{i,1}$,

$$\text{(D-2)} \qquad K_{i,1}^P \; = \; \left(1-\left(\sigma\ \alpha_{i,1}\right)/\left(1-CIS_i^P\right)\right) K_i^P \,.$$

Fourth, we compute the extent to which the sum of the country capital stocks in the first iteration ($\overline{K}_1^P$) deviates from the fixed amount of global private capital ($\overline{K}^P$),

$$\text{(D-3a)} \qquad \Delta_2 \equiv \left(\overline{K}_1^P - \overline{K}^P\right)/\,\overline{K}^P,$$

$$\text{(D-3b)} \qquad \overline{K}_1^P \equiv \sum_i K_{i,1}^P \,,$$

$$\text{(D-3c)} \qquad \overline{K}^P \equiv \sum_i K_{i.0}^P \,,$$

$$\text{(D-3d)} \qquad \Gamma_2 = 1 + \Delta_2 \,,$$

where $K_{i.0}$ is the initial value of country i's private capital stock.

We begin the second iteration by using this deviation to compute a new estimate of the global optimum MPK,

$$\text{(D-4)} \qquad \widehat{MPK_2}^{**} = \Gamma_2 \ \widehat{MPK_1}^{**} \,,$$

return to step 1, and insert $\widehat{MPK}_2^{**}$ into equation (D-1). The process continues until Δ_N is sufficiently close to 0.

Convergence is assured. Consider, for example, the situation where the total amount of private capital consistent with $\widehat{MPK}_n^{**}$ in the nth iteration exceeds the fixed, global, private capital stock. In this case, Δ_{n+1} is positive, Γ_{n+1} is greater than one, $\widehat{MPK}_{n+1}^{**}$ rises, and $\bar{K}_{n+1}^P$ falls. Since the negative relation between $\widehat{MPK}_{n+1}^{**}$ and $\bar{K}_{n+1}^P$ is monotonic, the process is guaranteed to converge to a unique solution, $\left\{K_i^{P**},\ i=1,88\right\}$.

The change in output from this globally optimal allocation of private capital is computed with a discrete version of equation (18), weighted by country-specific output shares,

$$\text{(E-5)} \qquad \frac{d\bar{Y}^{**}}{\bar{Y}} = \frac{\sum_i \left(Y_i^{**} - Y_i\right)}{\sum_i Y_i} = \sum_i \left(C\, IS_i^P \frac{dK_i^{P**}}{K_i^P} \right) \frac{Y_i}{\sum_i Y_i}\,.$$

Appendix E: Computing The Gains From Capital Reallocations In Terms Of 2019 (First Quarter) U.S. Dollars

This appendix provides details about the computations of the gains from reallocating capital from rich to poor countries discussed in Section VIII. All dollar figures are in billions of U.S. dollars in 2019.1 (the first quarter of 2019). The Renminbi figure is in billions.

1. Increase in world GDP.

$1,047 = 0.012 [Table 7, panel A, column 1] * $87,270 [IMF-WEO]
$873 = 0.010 [Table 7, panel A, column 2] * $87,270 [IMF-WEO]
$1,309 = 0.015 [Table 7, panel A, column 3] * $87,270 [IMF-WEO]

2. United States' stimulus program, The American Recovery and Reinvestment Act (ARRA). Outlays occurred over several years. We assume all outlays were made in 2009.

$774 = $663 [CBO] * 1.168 [1 + percentage change in the GDP price deflator from 2009 to 2019.1, FRED-PGDP].

3. China's stimulus program, We assume all outlays are made in 2008.

$677 = 4,000 Renminbi [CGPY] / 6.9477 [Renimbi/Dollar exchange rate in 2008, FRED-ER] * 1.176 [1 + the percentage change in the GDP price deflator from 2008 to 2019.1, FRED-GDP].

LEGEND

CBO: Congressional Budget Office, "Estimated Impact of the American Recovery and Reinvestment Act on Employment and Economic Output in 2014," (February 2015), Table 1.

CGPY: Cong, William Lin, Gao, Haoyu, Ponticelli, Jacopo, and Yang, Xiaoguang, "Credit Allocation under Economic Stimulus: Evidence from China," Chicago Booth (November 2018), p. 1.

FRED-ER: Federal Reserve Economic Data, "China / U.S. Foreign Exchange Rate (DEXCHUS)," https://fred.stlouisfed.org/series/DEXCHUS#0 .

FRED-PGDP: Federal Reserve Economic Data, "GDP Implicit Price Deflator in United States (USAGDPDEFAISMEI)," https://fred.stlouisfed.org/series/USAGDPDEFAISMEI .

IMF-WEO: International Monetary Fund, "IMF DataMapper," https://www.imf.org/external/datamapper/NGDPD@WEO/WEOWORLD .