Trade Policy-Making in a Model of Legislative Bargaining

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Abstract

In democracies, trade policy is the result of interactions among many agents with different agendas. In accordance with this observation, we construct a dynamic model of legislative trade policy-making in the realm of distributive politics. An economy consists of different sectors, each of which is concentrated in one or more electoral districts. Each district is represented by a legislator in the Congress. Legislative process is modeled as a multilateral sequential bargaining game à la Baron and Ferejohn (1989).

Some surprising results emerge: bargaining can be welfare-worsening for all participants; legislators may vote for bills that make their constituents worse off; identical industries will receive very different levels of tariff. The results pose a challenge to empirical work, since equilibrium trade policy is a function not only of economic fundamentals but also of political variables at the time of congressional negotiations.

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1 Introduction

“But to introduce a tariff bill into a congress or parliament is like throwing a banana into a cage of monkeys. No sooner is it proposed to protect one industry than all the industries that are capable of protection begin to screech and scramble for it. They are, in fact, forced to do so, for to be left out of the encouraged ring is necessarily to be discouraged.” – Henry George (1886).

Attempts by economists to understand the process of trade policy formation\(^1\) have evolved from approaches based on electoral competition (Mayer, 1984), through lobbying (Findlay and Wellisz, 1982) and influence-peddling (Grossman and Helpman, 1994), to more recent work focussed on the workings of legislative assemblies (such as Grossman and Helpman, 2005 and Willmann, 2004).

In this paper, we add an important element to the analysis: dynamic, non-cooperative congressional bargaining. Models that focus on congressional decision making assume a unified majority party writes and passes a bill, such as Grossman and Helpman (2005), or that implicit cooperative congressional bargaining maximizes joint utility of representatives, as in Willmann (2004). These approaches provide simplicity by clearing away by assumption many of the features that make trade policy complicated in practice. By contrast, in the model we propose here, in order to move tariffs from the status quo, the member of the Congress who can set the agenda must propose a trade bill and find a majority coalition willing to support it. In choosing how to vote, each member considers the uncertainty over who will have agenda-setting power next, and thus over what tariff bill will emerge down the road if the current bill fails. In this setting, a number of features emerge that are quite different from what other models offer:

(i) The trade policy that emerges will depend on which member of the Congress has agenda-setting power, apart from the fundamentals generally accounted for in empirical

\(^1\)There is an extensive literature on trade policy formation; see Rodrik (1995) and Nelson (1999) for a review.
work—both economic fundamentals (industry size, elasticities of demand, and so on) and institutional fundamentals (political organization of the industry). Indeed, since omnibus trade bills are passed infrequently, this implies that, at any given date, the structure of tariffs across industries can be largely the result of who chaired what committee, for no matter how brief a period, many years ago.

(ii) The equilibrium of the bargaining game is typically in mixed strategies. Conditional on fundamentals and the identity of the agenda-setter, the outcome is random because the agenda setter chooses randomly between industries to attract to the winning coalition. The randomness of this choice is a deep feature of the model that results from the dynamic nature of the game—specifically, the possibility of multiple rounds of bargaining after the current round if no bill passes; a static game would have no reason for randomness. As a result, even conditional on fundamentals and also on the identity of congressional leadership at the time of the bill’s passage, empirical work explaining the determinants of tariffs might need to control for the identity of the winning coalition, perhaps proxied by the members who voted in favor.

(iii) Because of the uncertainty about the future agenda setter and future proposals that will come to the table, in many cases a member of the Congress will vote for a bill that is worse for her constituents than the status quo. Again, this is a feature of the dynamic nature of the model and would disappear in a static version.

The key elements of the model can be seen in the rough and tumble of trade-bill formation in practice. Consider the 1880’s in the United States, a period in which trade policy was perhaps the most contentious and vigorously debated issue of the day and the issue on which at least one national election was decided. Consider three central elements to our story, each of which requires a dynamic model. (i) Uncertainty about the agenda setter. The main agenda setter in the US House of Representatives for trade bills is the chair of the Ways and Means Committee, appointed by the Speaker of the House. In 1883, the Speaker was Samuel Randall, a Democrat from Pennsylvania, an ardent protectionist allied with iron and other industries of his home state. He was challenged in that year for the position of Speaker by
John Carlisle of Kentucky, a Democrat from a rural area committed to much lower tariffs all around. The battle for the chairmanship was intense, and Carlisle surprised everyone by pulling an upset victory (Tarbell, 1911, p. 137). Carlisle appointed a moderate free-trader, William Morrison of Illinois, to the Chairmanship of Ways and Means. Later, the agenda-setter changed unpredictably once again, when Morrison in 1886 lost his re-election campaign and was replaced at Ways and Means by a stauncher free trader, Roger Mills of Texas (Tarbell, 1911, p. 155). The agenda-setter changed more dramatically in 1888, when Republicans won a majority in the House, and staunch protectionists seized control from the ardent free traders. Thus, in just a few years, the agenda-setting power changed hands several times, among politicians with very different policy preferences. (ii) The proposed trade bill changes dramatically with the identity of the agenda setter. Several different trade bills were proposed during this period, and the proposed bills changed character rapidly with changes in the agenda setter. For example, the Mills bill of 1888 lowered tariffs across the board, while the McKinley tariff bill passed by the new Republican house in 1890 raised tariffs sharply for almost every industry (Tarbell, 1911, pp. 188-206). (iii) The coalition supporting a proposed bill does not depend merely on party, but on the contents of the bill. Randall, for example, had built a coalition of supporters for his protectionist agenda that included a wide range of Republicans and a number of Democrats willing to buck their party’s dominant free-trade ideology (Tarbell, 1911, p. 137). (iv) Members of the Congress vote strategically, sometimes voting for a proposal that will make things worse for their constituents, because they are concerned that the next proposal might be even worse. This can be seen in the decision by Randall’s supporters in the House to support the Mills tariff reduction bill ‘with heavy hearts’ as likely the best they could obtain, although it reduced tariffs rather than raised them as their constituents desired (Tarbell, 1911, pp. 164-165).

Thus, the main elements of our model are very much in play in the real world of congressional tariff-setting. Now, to sketch our model in more detail. We consider a small open economy that accommodates four industries, one that produces numeraire homogeneous good using labor alone and three non-numeraire manufacturing industries that employ
sector-specific capital alone. There are $N$ electoral districts (constituencies), each of which hosts one manufacturing industry along with the numeraire good industry. Individuals who reside in the same district are identical and each one is endowed with one unit of labor and one unit of capital to be used in the manufacturing industry located in that district. As a result, there is a potential conflict among districts based on (manufacturing) industry attachment.\footnote{Magee (1978), in his study of testimony on trade legislation, finds strong evidence for sector-based political activity. Moreover, in other studies, capital and labor are found to be relatively immobile over politically plausible time horizons; see Nelson (2007), footnote 4.}

Each district is represented by a legislator in the legislature (Congress). Each legislator cares only about the welfare of her own district, and the welfare of a district is closely related to the industry located in it. In our model, trade policy implies any tariff or subsidy levied on any sector’s output. This setup is consistent with distributive politics since an increase in the price of a particular good (say, due to protection) will be beneficial only to those districts that produce it, but will be costly to the whole economy due to its negative effect on consumption.

We analyze the legislative game as a sequential model of multilateral bargaining with a simple majority rule à la Baron and Ferejohn (1989). This approach to congressional bargaining has borne much fruit in the political economy of public finance (see, among others, Baron, 1993; Primo, 2006 and Battaglini and Coate, 2007), but to our knowledge it has not yet been used to analyze trade policy. Each period, a proposer is selected randomly among risk-neutral legislators to propose a trade policy.\footnote{We do not model the election of legislators in this paper. However, the results would not change as long as there is no commitment to party loyalty.} To pass a bill, the proposer must

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\begin{itemize}
  \item[2] Constituencies are usually described in geographic terms in studies of the Congress; see Anderson and Baldwin (1987), Irwin and Kroszner (1999) and Lindsay (1990). This is intuitive since the electorate for members of Congress is defined geographically.
  \item[3] Magee (1978), in his study of testimony on trade legislation, finds strong evidence for sector-based political activity. Moreover, in other studies, capital and labor are found to be relatively immobile over politically plausible time horizons; see Nelson (2007), footnote 4.
  \item[4] We do not model the election of legislators in this paper. However, the results would not change as long as there is no commitment to party loyalty.
  \item[5] This is in line with the empirical literature; see, among others, Baldwin (1976; 1985), Hiscox (2002), Fordam and McKeown (2003) and Ladewig (2006). The underlying assumption is that each legislator must calculate how voters in her district will respond to a particular trade policy in order to maximize her probability of re-election.
  \item[6] Here, we use tariffs as a measure of protection. In reality, non-tariff barriers (NTBs) are also used and very closely related to tariffs as documented by Ray (1981) and Marvel and Ray (1983).
  \item[7] Random recognition is a convenient way to model the uncertainty that legislators face, i.e., they do not know exactly which coalitions will form in the future if the current coalition fails to enact the legislation.
\end{itemize}
create a coalition of supporting legislators large enough to form a majority, in which case the program goes into effect and the legislature adjourns. Otherwise, the status quo trade policy prevails and the process is repeated with a new legislator (possibly the same as in the previous period). In her voting, a legislator compares the benefits accruing to her district from the current proposal to the value of continuing to the next stage.

A new trade policy affects individuals’ welfare through three distinct channels. First, all individuals earn capital rent through the sector-specific capital they own. A higher tariff for a particular good, therefore, benefits those individuals who have a stake in that industry. Second, individuals derive utility from consumption. Thus, a higher tariff on any good lowers their consumer surplus. Finally, individuals share the revenue (positive or negative) from all taxes and subsidies imposed through trade policy. Keeping these three effects in mind will be helpful in interpreting the results.

A closer look at our findings yields the following observations. First, the ex ante expected benefit an industry receives from congressional bargaining is affected by the industry’s dispersion (i.e., the number of electoral districts an industry operates in). To focus on the pure dispersion effect, consider the thought experiment in which each industry produces the same output, but they differ in the number of districts in which they operate. We show first that, independent of other factors, trade protection is higher for industry $i$ than industry $j$ if industry $i$ representatives constitute a majority in the Congress. This makes sense; if an industry is dispersed enough to have a majority representation in the Congress, then it will receive more protection due to its agenda-setting power.

Although the purely random selection is of course an abstraction, the uncertainty regarding the agenda setter is important in practice, as illustrated by the historical example discussed above, and this convenient abstraction is used in an enormous literature following Baron and Ferejohn (1989).  

Busch and Reinhardt (1999) argue that geographical concentration is different from political concentration. Our focus is on industry spread across political districts, hence more in line with the definition of political concentration. However, for simplicity, we do not distinguish between these two terms.

We do not have political parties in our model, so the only way an industry can hold sway over a majority of districts is to have voters with an economic stake in that industry residing in a majority of districts. If we had parties in the model, it could also be possible for an industry to gain influence by being concentrated in districts held by the majority party, even if the industry is geographically concentrated. Evidence for this is provided by Fredriksson, Matschke and Minier (2011), who show, using U.S. tariff data from 1993, that an industry receives positive protection if it is relatively concentrated in districts held by the majority party. An extension to allowing for parties could be of interest, but is beyond the scope of the present paper.
However, we show that if no industry has a majority in the Congress, more disperse industries have no advantage *ex ante* over less disperse industries. The reason is that a more disperse industry has a better chance of holding agenda-setting power (since it controls more seats), and so it can drive a harder bargain *when it is in a coalition*. Consequently, when it is not the agenda setter, it has a much lower probability of being included in a coalition. This is a subtlety that as far as we know has not been investigated in empirical work.

Second, in case no industry has majority representation in the Congress, the *ex ante* expected benefit an industry receives from Congressional bargaining is determined by that industry’s total output. In particular, larger industries that produce more output tend to benefit less from congressional negotiations over tariffs than smaller industries. The reason is that such an industry will generate fewer imports (since it will satisfy more of domestic demand from domestic production), and so the tariff revenue produced by a given tariff will be small; but this means that if a large industry is a member of the coalition that forms the tariff bill, the coalition partner will receive little benefit from a tariff on the large industry, and so will be unwilling to agree to a high tariff.

Third, in addition to these factors, the *status quo* tariffs also matter. In particular, if the initial protection for an industry is already high compared to other industries, then there is less room for that industry to improve over its *status quo* welfare since it is closer to its ideal protection level than others.

Our model is closely related to Willmann (2004), McLaren and Karabay (2004) and Grossman and Helpman (2005). In Willmann’s (2004) citizen-candidate model, heterogeneous districts behave strategically such that they have a tendency to choose candidates that are more protectionist than the district median. From a regional perspective, voters prefer a positive tariff, ignoring the tariff costs imposed on other districts. In contrast, from a national perspective, these tariff costs are internalized and, therefore, each district’s representative has to make a compromise. Knowing this compromise, in each district, voters

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10 The *status quo* tariffs matter for welfare effects even though in the limiting case where the members of Congress are very patient, they do not matter for the final level of tariffs. The effect of historical patterns of protection on current protection is documented by Lavergne (1983) and Ray and Marvel (1984).
in the first stage (namely, regional election stage) choose someone who is more protectionist than the median voter of that district.

McLaren and Karabay (2004) extend the standard median-voter framework to a rudimentary model of a government by assembly where parties compete by making binding election promises. In their model, the equilibrium tariff turns out to be the optimal tariff of the median voter in the median congressional district. They show that import-competing interests are more likely to receive protection if they are moderately geographically concentrated (neither too concentrated nor too dispersed). They also conclude that majoritarian systems tend to be more protectionist than presidential systems.

Grossman and Helpman (2005) consider a three-stage game. First, parties announce their policy platforms, then elections take place, and in the final stage, a particular trade policy is adopted. In this model, the lack of full commitment to announced party policies creates protectionist bias as long as districts differ in their capital endowments.

Willmann (2004) models the trade policy determination as a joint welfare maximization of all legislators whereas Grossman and Helpman (2005) model it as a joint welfare maximization of majority party legislators. On the other hand, McLaren and Karabay (2004) employ an election framework in which trade policy is predetermined. The common property of all of these papers is that there is not much scope for legislative procedures. In contrast, non-cooperative legislative bargaining is the core force behind trade policy formation in our model.

The rest of the paper is organized as follows. In the next section, we describe the basic model. In section 3, equilibrium is characterized. We discuss possible extensions in section 4. Section 5 concludes the analysis. All proofs are relegated to the Appendix.

2 Model

Consider a small open economy populated with a unit measure of individuals living in $N$ districts (where $N \geq 3$ and divisible by 3). There are $M = 4$ industries: one that supplies a homogeneous numeraire good (good 0) produced with labor alone, and three others, each
of which supplies a homogenous manufacturing good (goods 1 through 3) produced with sector-specific capital alone.\textsuperscript{11} In particular, we assume that the production technology for good 0 yields 1 unit of output per unit of labor input, and the technology for each non-numeraire good takes the following form: \( f_i(K_i) = \theta K_i \), where \( K_i \) and \( \theta \) denote the amount of the sector-specific capital used in sector \( i \) and the economy-wide productivity parameter, respectively.

Each district is composed of a homogeneous population; each individual residing in a given district is endowed with one unit of labor and also one unit of the same type of sector-specific capital.\textsuperscript{12, 13} Let the number of districts producing good \( i > 0 \) be denoted by \( n_i \) such that \( n_1 + n_2 + n_3 = N \). Without loss of generality, we assume that \( n_1 \geq n_2 \geq n_3 \). Districts that produce the same manufacturing good are populated by the same number of individuals. To save on notation, we let \( \mathcal{L}_i \) denote both the total amount of type-\( i \) capital in a type-\( i \) district and the total number of individuals residing in a type-\( i \) district. Given that the population is of unit mass, \( \sum_{i=1}^{3} n_i K_i = 1 \).\textsuperscript{14} Let \( q_i \) denote the amount of good \( i \) produced in a district that hosts industry \( i \), and \( Q_i \) denote the total amount of good \( i \) produced in the economy. Therefore, we have \( q_i = \theta K_i \) and \( Q_i = n_i q_i \).\textsuperscript{15} In addition, let \( p_i^* \) and \( p_i \) represent, respectively, the exogenous world price of good \( i \) and its domestic price. On the other hand, the numeraire good, good 0, has a world and domestic price equal to 1 (see footnote 18). Thus, the total rent that accrues to capital in district \( i \) is \( p_i q_i = \theta p_i K_i \), and the total labor

\textsuperscript{11} We limit the number of non-numeraire manufacturing industries to three for simplicity, as in Grossman and Helpman (2005). Results remain qualitatively similar for \( M > 4 \). There are, however, many more possibilities (of coalition formation) to consider. See section 4 for further remarks.

\textsuperscript{12} Our results carry over even if more than one industry is allowed in each district as long as each resident still holds only one sector-specific capital and in every district there is one industry with majority representation. This is true since each legislator will follow the interests of the median voter, who belongs to a particular industry under the conditions assumed here.

\textsuperscript{13} We do not model the location choice of a particular industry, rather we take it as given. However, we acknowledge that this choice may depend on the political influence they can exert in each location.

\textsuperscript{14} We allow only those districts that produce different goods to differ in the number of citizens residing. This is done to simplify the notation. Alternatively, it is possible to allow each district (even the ones producing the same good) to be populated by different number of individuals. All of our results continue to hold.

\textsuperscript{15} To make things simple and analytically tractable, aggregate output of each industry is perfectly inelastic in our setup. This is merely to eliminate some complexity, but there is some evidence that supply elasticities tend to be quite low in practice; see Marquez (1990) and Gagnon (2003).
income earned in district $i$ is $K_i$.

Each individual has an identical additively separable utility function given by:

$$u = c_0 + \sum_{i=1}^{3} u_i(c_i),$$

where $c_0$ is the consumption of good 0 and $c_i$ represents the consumption of good $i = 1, 2, 3$. We assume that $u_i(c_i) = R_i c_i - (c_i^2/2)$, where $R_i > 0$ and assumed to be sufficiently large.\(^{16}\) With these preferences, the domestic demand for good $i > 0$, implicitly defined by $u_i'(d(p_i)) = p_i$, is given by $d(p_i) = R_i - p_i$. The linearity of demand is not crucial for the main results of our paper, but it simplifies the analysis and permits a closed-form solution. The indirect utility of an individual with income $y$ is $y + s(p)$, where $p = (p_1, p_2, p_3)$ is the vector of domestic prices, and $s(p) = \sum_{i=1}^{3} [u_i(d(p_i)) - p_i d(p_i)]$ is the resulting consumer surplus.

Each district is represented by a single legislator who is concerned only with the welfare of her own district. A district’s welfare is the aggregate utility of all individuals in that district, which is equal to the total income plus the district’s share in total consumer surplus and total tariff revenue (or subsidy cost) for each good. Hence, a district that produces good $i$ has a welfare (for $i \neq j \neq k$):

$$W_i(p) = K_i + p_i \theta K_i + K_i \sum_{l=i,j,k} \frac{(R_l - p_l)^2}{2} + K_i \sum_{l=i,j,k} [(p_l - p_l^*) (R_l - p_l - Q_l)],$$

where the first term is the district’s labor income (equal to one unit of good 0 output per person), the second term is the capital rent, the third term is the consumer surplus captured by that district, and the last term is the share of tariff revenue (or subsidy cost).\(^{17}\) In addition, we denote $w_i(p)$ as the welfare of an individual with a stake in industry $i$, hence $W_i(p) = K_i w_i(p)$.

\(^{16}\)To be more precise, we require $R_i > p_i^* + \theta - Q_i$. This ensures that demand for good $i$ is positive at all prices that may occur in legislative bargaining. We also require $p_i^* \geq Q_i$ for each price to be positive. See section 3 for the determination of optimal prices.

\(^{17}\)We assume that tariff revenue (or subsidy cost) is distributed equally as a lump-sum transfer to each individual.
We consider an infinite-horizon model. Every period, there is a set of prices at which individuals make their production and consumption decisions, and enjoy the resulting welfare. The legislature can change the prevailing status quo, \( p^s = (p_1^s, p_2^s, p_3^s) \), by changing the domestic price of any good via legislative bargaining. We restrict the set of policy instruments available to the legislature and allow only for trade taxes and subsidies. A domestic price in excess of the world price implies an import tariff for an import good and an export subsidy for an export good. Domestic prices below world prices correspond to import subsidies and export taxes.\(^{18}\)

The timing of the trade-policy formation game in our model is based on the Baron-Ferejohn bargaining framework. This is a game of complete information. At the start of each period (before any production or consumption takes place), a legislator is selected randomly to propose a vector of domestic prices for all manufacturing goods produced,\(^{19}\) with equal probability for each legislator. Therefore, the probability that the proposer represents industry \( i \) is equal to \( \frac{n_i}{N} \). We restrict each domestic price to be non-negative and below some finite upper bound.\(^{20}\) If the proposal receives a simple majority, it is immediately implemented and the legislature adjourns. Each district’s welfare thereafter is evaluated at the new prices.\(^{21}\) If the proposal does not receive a majority, the process is repeated with another legislator randomly selected to propose a program. Bargaining continues until a program is implemented.\(^{22}\) Districts continue to receive their status quo welfare in every

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\(^{18}\)Without loss of generality, we assume that the tariff/subsidy on good 0 is equal to 0. Any tariff vector \( t' \) yielding domestic prices \( p^t + t' \) with \( t'_0 \neq 0 \) can be replaced by \( t'' = t' - \frac{t'_0}{p_0} \) without changing relative prices or any real values. Given that good 0 is the numeraire, this implies that \( p^t_0 = p_0 = 1 \).

\(^{19}\)To simplify, we assume that a period in the legislative game coincides with a production/consumption period.

\(^{20}\)To be more precise, we require each domestic price to satisfy: \( 0 \leq p_i < \bar{p}_i \), where \( \bar{p}_i = p^t_i + \frac{(R_i - p^t_i)^2 + (Q_i - \theta)^2}{2(\theta - Q_i)} \). These limits ensure that we get an interior solution in prices.

\(^{21}\)Note that once a proposal is accepted, the game ends so that there will be no future proposals. The underlying assumption is that the legislative bargaining process is too costly to be repeated. Indeed, it is possible to show that if a new proposal is allowed in every period even after an agreement is reached, then the expected welfare of each industry is lower. See the discussion section for more on this issue.

\(^{22}\)We model the legislative process as an infinite horizon game. According to Osborne and Rubinstein (1994), “a model with an infinite horizon is appropriate if after each period players believe that the game will continue for an additional period, while a model with a finite horizon is appropriate if the players clearly perceive a well-defined final period.” In our setup, there is uncertainty about the exact time when the legislative period will end. Therefore, there is no well-defined final period in our game.
period until an agreement is reached.

There are a couple of things to note. First, it is straightforward to show that the aggregate welfare, \( W(p) = \sum_{i=1}^{3} n_i W_i(p) \), is maximized at the free trade prices of the three goods. Hence, if the prices were set by a central authority (such as a President), free trade would prevail forever. In the current paper, however, we are interested in policy-making by a legislature. This naturally introduces a conflict of interest among legislators since each legislator is selfishly interested in maximizing her own district’s welfare. Put another way, when proposing a program, each legislator weighs the marginal benefits and costs for her own district but ignores the negative externality imposed on others. Therefore, the resulting trade policy is inefficient for the economy as a whole.

Second, from (2), a manufacturing good affects (through its price) a district’s welfare via three channels. The first one, the rent that accrues to the specific factor, is present if that good is produced in that district. The second one is the consumer surplus attained from the consumption of that good. The last one is the tariff revenue (or subsidy cost) due to trade. The effect of price through the first channel is always positive whereas it is always negative through the second channel. Its effect through the third channel, on the other hand, can be positive or negative (in fact the third channel is concave in all three prices). This is true since good \( i \)'s price has two distinct effects on tariff revenue/subsidy cost: (1) the direct effect (changing price while keeping imports/exports constant), and (2) the indirect effect through demand. These two effects work in opposite directions. To see this, assume that good \( i \) is an imported good. First, start from a price just above the world price. As we increase the price, the direct effect leads to an increase in the tariff revenue whereas the indirect effect leads to a decrease (since import demand goes down). Initially, the direct effect dominates, and therefore, raising the price raises tariff revenue. When the price reaches a certain value, the indirect effect starts dominating and the tariff revenue decreases if we further increase the price.
3 Characterization of equilibrium

In this section, we investigate the properties of the bargaining outcome. As common in multi-person bargaining problems, there may be many subgame perfect equilibria (SPE) in this game.\(^{23}\) We focus on stationary subgame perfect equilibrium (SSPE) whereby the continuation payoffs for each structurally equivalent subgame are the same.\(^{24}\) In a stationary equilibrium, a legislator who is recognized to make a proposal in any two different sessions behaves the same way in both sessions (in the case of a mixed-strategy equilibrium, this generally means choosing the same probability distribution over offers). Hence, stationary equilibria are history-independent. To make our results as clear as possible, we focus on the case in which the discount factor (denoted by \(\delta\)) approaches 1 in the limit.\(^{25}\)

Let the per-period equilibrium welfare of a district producing good \(i\), evaluated at the beginning of a period, before the proposer has been selected, be denoted as \(V_i\). This is also the per-period equilibrium welfare a district expects in the event that the period ends without a bill passed, and so we will also call it the ‘continuation payoff.’ (Recall that we are focussed on the limiting case as \(\delta \to 1\).) When faced with a proposal, a legislator votes yes only if the welfare her district gets in the current proposal is not less than its continuation welfare.\(^{26}\) Since a random proposer is selected every period, the outcome of legislative bargaining depends on the identity of the proposer. In this sense, the outcome is ex ante uncertain. We use the expected per-person welfare change due to bargaining as the basis for comparison among individuals with stakes in different sectors. To this purpose, let \(v_i\) denote the equilibrium continuation welfare of an individual with a stake in industry \(i\), thus \(V_i = K_i v_i\).

\(^{23}\)Baron and Ferejohn (1989) show that any outcome (in their game that means any division of the dollar) can be supported as an SPE using infinitely nested punishment strategies as long as there are at least five players and the discount factor is sufficiently high. Li (2009) shows that even with three players, there is a vast multiplicity of SPE.

\(^{24}\)Baron and Kalai (1993) argue that stationarity is an attractive restriction since it is the “simplest” equilibrium such that it requires the fewest computations by agents.

\(^{25}\)This corresponds to the case in which the time length between any two offers (sessions) is infinitesimally short. See section 4 for the implications of relaxing this assumption.

\(^{26}\)We assume that a legislator who is indifferent between accepting a proposal and continuing to the next session accepts the proposal. Therefore, weakly dominated strategies are ruled out.
The welfare of a district producing good $i$, given in (2), can conveniently be decomposed into two parts: the welfare evaluated at the \textit{status quo} prices, and the change in welfare due to legislative bargaining. Let $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$, where $\tau_i = p_i - p_i^*$. If good $i$ is an imported good and $p_i > p_i^*$ ($p_i < p_i^*$), then $\tau_i$ describes import tariff (subsidy). If, on the other hand, good $i$ is an exported good and $p_i > p_i^*$ ($p_i < p_i^*$), then $\tau_i$ describes export subsidy (tax).

Similarly, let $\boldsymbol{\tau}^* = (\tau_1^*, \tau_2^*, \tau_3^*)$ describe the vector of \textit{status quo} trade taxes (or subsidies). Noting that $\pi_i = \pi_i - \pi_i^* + \pi_{\tau\pi}i$, the welfare of a district that produces good $i$, given in (2), can be rewritten on a per capita basis as

$$w_i = 1 + \theta(\tau_i - \tau_i^* + p_i^*) + \sum_{l=i,j,k} \left( \frac{(R_l - (\tau_l - \tau_l^* + p_l^*) - Q_l)^2}{2} \right) + \sum_{l=i,j,k} \tau_l(R_l - (\tau_l - \tau_l^* + p_l^*) - Q_l)$$

$$= \left[ 1 + \theta p_i^* + \sum_{l=i,j,k} \left( \frac{(R_l - p_l^*)^2}{2} \right) + \sum_{l=i,j,k} \tau_l^*(R_l - p_l^* - Q_l) \right]$$

$$+ \left[ \theta(\tau_i - \tau_i^*) + \sum_{l=i,j,k} \left( \frac{(\tau_l - \tau_l^*)^2}{2} - 2(\tau_l - \tau_l^*)(R_l - p_l^*) + \sum_{l=i,j,k} (\tau_l - \tau_l^*)(R_l - (\tau_l - \tau_l^* + p_l^*) - Q_l) \right) \right],$$

where the first bracket is the welfare evaluated at the \textit{status quo} prices. Rearranging the term in the second bracket, the above expression becomes

$$w_i(\boldsymbol{\tau}) = w_i(\boldsymbol{\tau}^*) + \left( \frac{\theta(\tau_i - \tau_i^*) - \frac{1}{2} \sum_{l=i,j,k} [(\tau_l + Q_l)^2 - (\tau_l^* + Q_l)^2]}{2} \right). \quad (3)$$

This decomposition is helpful as it allows us to express the per capita welfare change in each district as a function of each industry’s trade tax and total output. The first term in the parentheses indicates the per-capita change in capital rent while the second term indicates the per-capita change in consumer surplus plus tariff revenue.

The first-best for each legislator is to maximize her district’s welfare without any constraints. Note that maximizing aggregate district welfare $W_i(\boldsymbol{\tau})$ is equivalent to maximizing per-capita welfare $w_i(\boldsymbol{\tau})$. For a legislator representing industry $i$, let $\boldsymbol{\tau}_i = (\tau_i, \tau_j, \tau_k)$ denote the vector of trade taxes that the unconstrained maximization problem leads to, i.e., $\boldsymbol{\tau}_i = \arg \max \limits_{\tau} w_i(\boldsymbol{\tau})$. Maximizing (3) with respect to $\tau_i, \tau_j$ and $\tau_k$ yields the following lemma.
Lemma 1  Unconstrained maximization of \( w_i(\tau), i = 1, 2, 3, \) yields (for \( i \neq j \neq k \))

\[
\begin{align*}
\tau_{i}^{U_i} &= \theta - Q_i, \\
\tau_{j}^{U_i} &= -Q_j, \\
\tau_{k}^{U_i} &= -Q_k.
\end{align*}
\]

Thus, a recognized (selected) legislator would ideally demand an import tariff (or an export subsidy) for the good her district produces (thereby protecting industry \( i \)) whereas an import subsidy (or an export tax) for the other goods. Moreover, a producer in a sector that produces higher aggregate output \( Q_i \) will prefer a lower tariff (or export subsidy) for her own product than a producer in a sector that produces lower aggregate output. The reason is as follows. Focus for now on the case of an imported good. Recall the three channels discussed before through which the tariff affects the per capita welfare of producers in industry \( i \). Aggregate output, \( Q_i \), in this case does not affect the first two channels (the rent and consumer surplus channels – of course, a higher \( Q_i \) implies higher total rent, but not higher rent per capital owner in industry \( i \)). What it does affect is the third channel, tariff revenue. A higher value for \( Q_i \) implies a weaker tariff revenue effect since, at a given price and the other parameters, a higher value of \( Q_i \) implies fewer imports, hence a lower marginal tariff revenue for a given increase in tariff.\(^{27}\) Therefore, a higher value of \( Q_i \) implies a lower marginal benefit of the tariff, and a lower optimal tariff, from the point of view of a sector-\( i \) producer. Parallel reasoning holds for an exported good.

It is natural to assume that the status quo prices are in the range defined by the unconstrained maximization problem. For example, a legislator representing a district that produces good \( i \) has no reason to set \( \tau_i \) above \( \theta - Q_i \). Similarly, she has no reason to set \( \tau_{j \neq i} \) below \( -Q_j \). Hence, we make the following assumption.

\(^{27}\)The same conclusion holds for a comparison between two industries \( i \) and \( j \) even if, although \( Q_i > Q_j \), the demand parameter \( R_i \) is sufficiently higher than \( R_j \) that at a common tariff, imports of good \( i \) exceed those of good \( j \). The reason is that an increase in \( R_i \), holding all prices and other parameters constant, raises industry \( i \) imports, increasing the marginal tariff revenue from the tariff on good \( i \), but at the same time raises domestic consumption of good \( i \), raising the marginal consumer surplus loss from the tariff on good \( i \). The two effects cancel each other out, with the result that the demand parameters \( R_i \) have no effect on tariff preferences.
**Assumption 1.** The status quo prices satisfy the following: 
\[-Q_i \leq \tau_i^s = p_i^s - p_i^* \leq \theta - Q_i, \]
for \(i = 1, 2, 3,\)

Hence, a value of \(\tau_i^s = \theta - Q_i\) corresponds to the case in which the status-quo tariff of good \(i\) is at its optimum for the districts that produce good \(i\), while \(\tau_i^s = -Q_i\) corresponds to the case in which it is at its optimum for the districts that produce good \(j \neq i\). Accordingly, the status quo corresponds to the optimal tariff vector for the districts that produce good \(i\) when \((\tau_i^s, \tau_j^s, \tau_k^s) = (\theta - Q_i, -Q_j, -Q_k)\).

As mentioned earlier, a proposal will be accepted when majority support is obtained in the Congress. As a result, there are two possible cases to be considered. We first analyze the situation when one of the manufacturing goods is sufficiently dispersed across the country so that the districts producing it have a majority representation in the Congress \((\frac{n_1}{N} > \frac{1}{2})\).

We next turn attention to a more even distribution of industries in which no manufacturing good has a majority representation in the Congress.

### 3.1 Case 1: \(\frac{n_3}{N} \leq \frac{n_2}{N} < \frac{1}{2} < \frac{n_1}{N}\)

This is the case in which one of the manufacturing goods has a majority representation in the Congress. The solution here is rather trivial; since we focus on the case in which all players are very patient \((\delta \to 1)\), legislators of districts producing good 1 would refuse any offer until they receive their first-best payoffs. In other words, districts that produce good 1 have the ultimate power in setting the prices of all three goods. When a legislator representing a district producing good 2 or good 3 is selected as the proposer, depending on the status quo welfare her district enjoys, she may choose to make a proposal that would be rejected by the legislature. However, in any SSPE, the accepted price vector must be \(\tau^{U_1}\).

**Proposition 1** When \(\frac{n_1}{N} > \frac{1}{2}\), the unique SSPE is described as follows.\(^{28}\) When a legislator representing a district that produces good 1 is selected to make a proposal, she proposes \(\tau = \tau^{U_1}\), and it is accepted immediately. When a legislator representing a district that produces

\(^{28}\)Note that under this case, the unique SSPE is also the unique SPE.
good $j = 2, 3$ is selected to make a proposal, she proposes $\tau = \tau^{U_j}$ if $w_j(\tau^s) \leq w_j(\tau^{U_j})$; otherwise she makes a proposal that would be rejected by the legislature.\footnote{Note that for each status quo price vector, there is a critical value of $\delta$ below which an agreement is always reached in the first period independent of the identity of the proposer. In contrast, the accepted price vector in this case depends on the identity of the proposer.} The equilibrium continuation payoffs are $v_i = w_i(\tau^{U_j})$.\footnote{These are the continuation payoffs in case of an agreement. Since an agreement can be reached only when the proposed prices are $\tau^{U_j}$, it follows that $v_i = w_i(\tau^{U_j})$.}

Here, industry 1 is sufficiently large (i.e., it has the necessary number of seats in the legislature) to set trade policy independently. As a result, the problem becomes an unconstrained maximization problem. By (3), the expected per-person welfare change can be expressed as

$$v_1 - w_1(\tau^s) = \theta \left[ (\theta - Q_1) - \tau^s_1 \right] - \frac{\theta^2 - \sum_{i,j,k} (\tau^s_i + Q_1)^2}{2},$$  
$$v_2 - w_2(\tau^s) = -\theta \left[ Q_2 + \tau^s_2 \right] - \frac{\theta^2 - \sum_{i,j,k} (\tau^s_i + Q_1)^2}{2},$$  
$$v_3 - w_3(\tau^s) = -\theta \left[ Q_3 + \tau^s_3 \right] - \frac{\theta^2 - \sum_{i,j,k} (\tau^s_i + Q_1)^2}{2}. \tag{4}$$

From (4), it is clear that for a given value of $\theta$, $v_i - w_i(p^s)$ is increasing in $\tau^s_{j\neq i}$ and, given Assumption 1, decreasing in $\tau^s_i$. For an individual who has a stake in industry $i$, a low value of $\tau^s_i$ corresponds to the case in which the status quo tariff for good $i$ is significantly different than its optimum value, and hence, there is room for welfare improvement. As $\tau^s_i$ increases, the potential welfare gain via the change in the price of good $i$ gradually diminishes and reaches zero when $\tau^s_i = \theta - Q_i$. This is the case in which the status quo tariff for good $i$ is already at its optimum for industry $i$ agents. A parallel argument can be made for $\tau^s_{j\neq i}$.

For an individual who has a stake in industry $i$, a high value of $\tau^s_{j\neq i}$ means that there is a big room for welfare improvement by lowering the price of good $j$. As $\tau^s_{j\neq i}$ goes down, the potential improvement via the change in the price of good $j$ becomes lower and reaches zero when $\tau^s_{j\neq i} = -Q_j$. Again, this is the case when the status quo tariff for good $j$ is already at its optimum for industry $i$ agents.

Furthermore, $v_i - w_i(p^s)$ is increasing in $Q_{j\neq i}$ and decreasing in $Q_i$. As stated before,
aggregate output of each industry affects individual welfare via the third channel, namely the tariff revenue effect. We know from Lemma 1 that each individual prefers a price above the world price for the good in which she has a direct stake and a price below the world price for the other goods. Consider imported goods for the moment. This implies that each individual receives a tariff revenue for its own industry’s good and incurs a subsidy cost for other goods. A higher value of $Q_i$ ($Q_{j\neq i}$) implies fewer imports, hence a lower tariff revenue (subsidy cost). A similar reasoning holds for exported goods.

Moreover, in light of the assumed range for the status quo tariffs, it is possible to rank the welfare change from the best to worst across individuals with stakes in different industries. Note that, for a given $\tau^s_2$ and $\tau^s_3$, $v_1 - w_1(\tau^s)$ attains its minimum at $\tau^s_1 = \theta - Q_1$. Evaluated at this value, $v_1 - w_1(\tau^s) = \frac{(\tau^s_2+Q_2)^2+(\tau^s_3+Q_3)^2}{2} \geq 0$, so there is always a welfare gain for individuals who have a stake in industry 1.\(^{31}\) On the other hand, for individuals associated with industries 2 and 3, whether there is a welfare gain or loss depends on the values of $\tau^s_i$, $\forall i$. Given that $-Q_i \leq \tau^s_i \leq \theta - Q_i$, it is easy to see that the maximum value of $[v_2 - w_2(\tau^s) + v_3 - w_3(\tau^s)]$ is zero, implying that at least one of the two must be negative (except for when $(\tau^s_1, \tau^s_2, \tau^s_3) = (\theta - Q_1, \theta - Q_2, \theta - Q_3)$ or $(\theta - Q_1, -Q_2, -Q_3)$, in which case both of them are zero). Moreover, if $\tau^s_2 + Q_2 > \tau^s_3 + Q_3$, then $v_2 - w_2(\tau^s) < v_3 - w_3(\tau^s)$ (and vice versa). However, per-capita welfare gain accruing to industry 1 is at least as much as any possible welfare gain accruing to other industries. This is due to the agenda-setting power of the legislators representing industry 1.

In short, in Case 1, the majority industry sets a positive tariff for itself and a negative tariff for all other industries, and does so without any strategic constraint since it needs no coalition partners or consent. Now we consider the more interesting case in which no manufacturing industry can control the majority of seats in the legislature, so a legislator who can propose a trade bill needs the support of the legislators representing at least one other industry. To get their votes, she has to offer them a more favorable tariff compared to the unconstrained case, which moves the final outcome away from her first-best. This is

\(^{31}\)When $\tau^s_1 = \theta - Q_1$, $\tau^s_2 = -Q_2$ and $\tau^s_3 = -Q_3$, the status quo tariffs coincide with the optimal tariffs; i.e., $\tau^s = \tau^{t^*_i}$. In this case, the welfare change will be zero.
what we analyze next.

### 3.2 Case 2: $\frac{n_i}{N} \leq \frac{n_j}{N} \leq \frac{n_k}{N} \leq \frac{1}{2}$

Here, we analyze the bargaining outcome when no industry is highly dispersed throughout the economy. In this case, unlike in Case 1, a recognized legislator may not be able to achieve the first-best outcome for her district since she has to compromise with at least one other industry in order to attain simple majority of the votes. We refer to this situation as the proposer selecting a ‘coalition partner’ (or simply ‘forming a coalition’).

When a legislator is recognized to make a proposal, she has an incentive to propose a program that will be accepted, since if rejected, she faces the risk that her district will be worse off in the program adopted in the future. In equilibrium, in accordance with the “Riker’s (1962) size principle,” any proposal will be accepted with the minimal number of industries that constitute a quorum of districts. This is true since increasing the number of industries in the coalition would increase the costs without increasing the benefits. Hence, the recognized legislator will make a proposal that will be accepted by not only the representatives of the districts producing the same manufacturing good but also the representatives of the districts producing one other good. In order to get the acceptance of the latter, the proposal should provide their districts an *ex post* welfare not less than their discounted continuation payoffs.

Suppose a legislator representing a district which produces good $i$ is recognized to make a proposal. If she chooses to reward the districts which produce good $j \neq i$, her maximization problem expressed on a per capita basis becomes:

$$\max_{\tau} w_i(\tau) \text{ s.t. } \frac{w_j(\tau)}{1-\delta} \geq \frac{w_j(\tau^*) + \delta v_j}{1-\delta}.$$ (5)

The left-hand side of the inequality indicates the discounted total welfare at the proposed prices, whereas the right-hand side is what a legislator representing a district that produces good $j$ expects in case she does not vote in favor of the proposal (individuals in her district obtain their *status quo* welfare for the current period and the expected continuation welfare
thereafter).\footnote{Note that districts that accommodate the same industry are identical, so if this inequality holds for one, then it also holds for all.} The recognized legislator will choose \( \tau \) such that the constraint is satisfied with equality. Thus, the constraint can be rewritten as:

\[
w_j(\tau) = (1 - \delta)w_j(\tau^s) + \delta v_j.
\]

As in Baron and Ferejohn (1989), in Case 2, in an SSPE with \( \delta \) close to 1, generically there is an equilibrium in which the proposer randomizes between the two other industries in choosing a coalition partner. The proof is in the appendix, but the crux of the idea can be summarized as follows. In an SSPE, by definition, if proposer \( i \) ever chooses industry \( j \) with probability 1, then (due to stationarity) she always will choose industry \( j \) with probability 1. But this means that industry \( j \) has enormous bargaining power, and consequently at any given date, it will be less attractive for \( i \) to choose \( j \) than the other industry – a contradiction. Let \( s \) denote the probability that \( i \) will choose \( j \), and hold constant the behavior of the other players when they are proposers. A reduction in \( s \) lowers \( j \)'s continuation value, hence bargaining power, and raises \( k \)'s \((i \neq k \neq j)\). Therefore, a critical value of \( s \) exists at which \( i \) is indifferent between the two potential coalition partners, and this is the equilibrium value. The proper proof must take into account boundary conditions as well as the fact that each player’s probability over partners is endogenous, and it turns out that when all three players’ probabilities are determined together, the equilibrium choice of probabilities is not unique, although the payoffs are. In the proof of Proposition 2, we first show that when \( \delta \to 1 \) an SSPE exists in which all legislators randomize between the other two industries. We then prove that all SPPE are payoff equivalent. We now present the main result.

\begin{proposition}
When \( \frac{\mu_i}{N} \leq \frac{1}{2} \), in the limit as \( \delta \to 1 \) an SSPE exists in which a selected legislator representing a district which produces good \( i \) proposes a tariff \( \tau_i = \frac{2}{3} \theta - Q_i \) for the good her district produces, a tariff \( \tau_j = \frac{1}{3} \theta - Q_j \) for good \( j \neq i \) where \( j \) is selected randomly, and a tariff \( \tau_k = -Q_k \) for the remaining good \( k \). The first proposal receives a majority vote, so the legislature adjourns after the first session. The ex post per-capita payoffs are

\[
w_i = w_i(\tau^s) + \theta \left[ \left( \frac{2}{3} \theta - Q_i \right) - \tau_i^s \right] - \frac{\mu_i^2 - \sum_{l=1,j,k} \left( \tau_i^l + Q_l \right)^2}{2} \text{for the industry the proposer represents,}
\]

\end{proposition}
\[ w_j = w_j(\tau^s) + \theta[(\frac{\theta}{T} - Q_j) - \tau^s_j] - \frac{5\theta^2}{\theta} - \frac{\sum_{i=1,j,k}[(\tau^s_i + Q_i)^2]}{2} \] for the industry the coalition partner represents, \( w_k = w_k(\tau^s) + \theta[-Q_k - \tau^s_k] - \frac{5\theta^2}{\theta} - \frac{\sum_{i=1,j,k}[(\tau^s_i + Q_i)^2]}{2} \) for the remaining industry that is outside the coalition.

Note also that since \( \delta \rightarrow 1 \), ex ante expected per capita payoffs for any industry \( i \) before the proposer is determined are the same as the ex post payoffs that industry would obtain if it was chosen ex post as the coalition partner, which is given by

\[ w_i = w_i(\tau^s) + \theta[(\frac{\theta}{T} - Q_i) - \tau^s_i] - \frac{5\theta^2}{\theta} - \frac{\sum_{i=1,j,k}[(\tau^s_i + Q_i)^2]}{2} \text{ for } i \in \{1, 2, 3\}. \]

Compared to Case 1, since the proposer needs the approval of one other industry, she compromises by proposing a lower price for her own industry and a higher price for the industry selected as the coalition partner. Below, we summarize the important properties of this SSPE.

1. For given values of \( \theta, \tau^s_i \) and \( Q_i \), \( \forall i \in \{1, 2, 3\} \), the continuation payoff of any district (or expected welfare change of any individual) is in between the highest and the lowest continuation payoffs obtained under Case 1. This makes sense since no industry is dispersed enough to control the legislature single-handedly. Therefore, no industry is either very strong or very weak. Thus, compared to Case 1, districts producing goods 2 and 3 have significantly higher bargaining power, which, in turn, reduces the welfare gain (may even result in welfare loss) districts that produce good 1 expect.

2. Per capita expected welfare change of each individual with a stake in industry \( i \) is increasing in \( \tau^s_{j \neq i} \) and \( Q_{j \neq i} \) and decreasing in \( \tau^s_i \) and \( Q_i \) as in Case 1 for exactly the same reasons stated before. Moreover, depending on the values of \( \tau^s_i \), the ex ante expected welfare change can be positive or negative for each industry, and it can be positive for all of them or negative for all of them.\(^{33}\) In contrast, in Case 1, industry 1 always observes a welfare gain, and, independent of \( \tau^s \), there is always one industry (must be either industry 2 or 3) which experiences a welfare loss.

\(^{33}\) For instance, when \((\tau^s_1, \tau^s_2, \tau^s_3) = (\theta - Q_1, \theta - Q_2, \theta - Q_3)\), \( v_i - w_i(\tau^s) = \frac{5\theta^2}{\theta} > 0 \) for all \( i \). Similarly, when \((\tau^s_1, \tau^s_2, \tau^s_3) = (\frac{\theta}{2} - Q_1, \frac{\theta}{2} - Q_2, \frac{\theta}{2} - Q_3)\), \( v_i - w_i(\tau^s) = -\frac{1}{20}\theta^2 < 0 \) for all \( i \).
3. The ranking of *ex ante* welfare gains for individuals with stakes in different industries depends only on the values of $\tau_i^a$ and $Q_i$, $\forall i \in \{1, 2, 3\}$ and are independent of industry dispersion (as long as $\frac{n_i}{N} \leq \frac{1}{2}$ for all $i$). This last point may be surprising: an industry that dominates twice as many congressional districts as another receives no net advantage from that fact (as long as it does not have a majority) — even though it will thereby have twice the probability that one of its representatives will be the proposer. The reason comes from the dynamic nature of the bargaining. If industry $i$ has a large minority of the seats and thus a high probability of being the proposer, it will gain from a high tariff if it is the proposer; and in any round where $i$ is not the proposer, the probability that it will become so in the next round is high. But the other representatives will understand that industry $i$ will therefore drive a tough bargain if another industry is the proposer and chooses $i$ as a coalition partner, so $i$ will rarely be chosen as a coalition partner. Industry $i$’s benefit from being the proposer with high probability is exactly cancelled out by its loss from being excluded from the coalition with high probability when it is not the proposer.\(^{34}\)

We should note that in the mixed strategy equilibria, randomization probabilities are not unique although they all lead to the same set of payoffs, as stated in the following proposition.\(^{35}\)

**Proposition 3** All SSPE are payoff-equivalent.

More broadly, Proposition 2 provides a number of characteristics for the equilibrium that are strikingly different from characteristics of models without dynamic bargaining and that may be useful in empirical work or in interpreting tariff history. First, note that the

\(^{34}\)This can be seen formally from the proof in the Appendix. Equation (A.2) shows how an industry’s *ex ante* expected benefit can be written in terms of probabilities of being the proposer ($\frac{N}{N}$) and the probability of being the coalition partner ($s_{ij} \frac{n_i}{N} + s_kj \frac{n_k}{N}$) in addition to parameters. The remainder of the proof shows that these probability terms cancel out, implying that any increase in $\frac{N}{N}$ and corresponding decrease in $\frac{N}{N}$ or $\frac{N}{N}$ results in adjustment of $s_{ij}$ and $s_kj$ (the probability that $j$ is picked by $i$ or $k$) so that the probability of being a coalition partner falls by $1/\lambda = 2$ times as much as the increase in $\frac{N}{N}$.

\(^{35}\)The same multiplicity is also present in the standard symmetric Baron-Ferejohn game, see Celik and Karabay (2011). Eraslan (2002) shows that all SSPE in the Baron-Ferejohn game are payoff equivalent when the recognition probabilities are asymmetric.
equilibrium tariffs are a function of economic fundamentals such as industry size $Q_i$, but they are also a function of political variables at the time of the congressional negotiation. Note that after controlling for industry size, the tariff is highest for the industry represented by the proposer and lowest for the excluded industry. The identity of the proposer is most plausibly determined by the party with the majority in the Congress at the time of the tariff bill together with internal party competition for the leadership post; years later, even with different leadership, the tariff structure will be determined partly by the political conditions at the time of the tariff bill. Even conditional on the identity of the proposer, the tariff structure is affected very much by the identity of the coalition partner, which receives a tariff premium, and this choice is necessarily randomized due to the mixed equilibrium required by the dynamic logic of the model. In empirical work, one might imagine a number of proxies for the ‘proposer,’ including, in the US case, the chairmanship of the House Ways and Means Committee or the Senate Finance committee; and one might think of using an ‘aye’ vote on the most recent tariff bill as a proxy for the theoretical construct of the ‘coalition.’ Both should have a significant correlation with tariffs.

Another way of looking at this is that the logic of congressional bargaining imposes different levels of protection for different industries even if all industries are ex ante identical. Suppose that $K_1 = K_2 = K_3$, $n_1 = n_2 = n_3$, $p^*_1 = p^*_2 = p^*_3$, and $p^*_1 = p^*_2 = p^*_3$. Then most other models would predict $t_1 = t_2 = t_3$. Grossman and Helpman (2005) would predict the same tariff for each industry within the same party. However, in our model, there would be three separate levels of tariff, which would be correlated with the legislators’ votes, and not correlated with party affiliation once the votes have been controlled for. Thus, the empirical predictions of this model are quite different from those of other models.

Second, note that often representatives in the Congress in this model will vote for a bill that they do not like, because with the dynamic bargaining, they are afraid that if the current bill does not pass, it will be replaced with something that they like even less. This can be seen clearly by examining point 2 above. It is easy to find parameters such that the ex ante welfare change resulting from the bargaining is negative for each industry. For
example, suppose that the status quo is free trade in a symmetric economy, so that $\tau_i^* = 0$ and $Q_i = \frac{\theta}{3}, \forall i \in \{1, 2, 3\}$. In this case, the ex ante expected welfare change for each industry is negative (as can be seen from the equation immediately after Proposition 2), since each industry knows that total welfare will fall as tariffs are introduced by the bargaining, but no-one knows who the ex post beneficiary will be. Consequently, the ex post welfare change for the coalition partner as a result of the bargaining will be negative. This implies that the coalition partner will vote for a tariff bill that lowers its utility relative to the status quo (in this case, a tariff bill that gives no tariff at all to its own products, while providing a positive tariff to the proposer and a negative one to the excluded industry). This is because it fears the possibility of being the excluded industry in the next round. It will support the bill as some members of the Congress from manufacturing districts supported the tariff-reducing Mills bill of 1888: “with heavy hearts” (Tarbell, 1911, p. 165).

Indeed, it is easy to find cases in which the proposer proposes and votes for a bill that lowers its utility relative to the status quo, because it is aware that it might not be the proposer in the next round and may face something worse. As an example, suppose that the status quo tariffs are close to the unconstrained optimum tariffs $\tau_i^{U_i}$ for industry $i$ from Lemma 1. In this case, industry $i$ knows that if it is not the proposer, those tariffs will be changed and it will lose utility, so it will cut its losses and find a tariff bill that its coalition partner will agree to now. This is in the same spirit as when protectionist Republicans, following a rousing speech in which President Cleveland made the case for free trade and the political momentum was moving in that direction, struggled to come up with a strategy for reducing tariffs in a way that would blunt that momentum: “Protection must be preserved. If its operations were to be corrected, this must be done by its friends, not its enemies.” (Tarbell, 1911, p. 154.)

4 Discussion

In this section, we discuss four points related to possible extensions of our model. The first one regards the number of industries. We have, for simplicity, considered only three
(manufacturing) industries. It is possible to generalize this to a larger number of industries. The main intuition still holds. If one industry has majority representation, then that industry gains the most. On the other hand, if none of the industries has a majority, then it is the total production and status quo tariff/subsidy that determine the gains for each industry. For example, consider four industries with the following distribution: \( \frac{n_1}{N} = 0.4, \frac{n_2}{N} = 0.3, \frac{n_3}{N} = 0.25, \frac{n_4}{N} = 0.05 \). In this example, industry 4 is too small to be valuable as a partner in any coalition. However, when welfare changes from bargaining are considered, it is still possible for industry 4 to benefit more than others as long as \( \tau_4 + Q_4 \) is small enough and \( \tau_i + Q_i \)'s for \( i = 1, 2, 3 \) are large enough. Moreover, assuming symmetric dispersion of industries, as the number of industries increase, the ex post tariffs (as well as the ex ante tariffs) decrease.\(^{36}\)

The second point is about the bargaining procedure. We have assumed that once an agreement is reached, bargaining ends. Instead, assume that legislators bargain every period and that if an agreement is reached in the previous period, it constitutes the status quo for the current period. In the context of a divide-the-dollar game, Kalandrakis (2004) shows that there is a Markov equilibrium in which, irrespective of the initial status quo payoffs, whoever is the proposer is eventually able to achieve her first-best (i.e., take the whole dollar). The intuition is as follows. In every period, players reach an agreement in which there is at least one player who receives nothing. Since this constitutes the status quo for the following period, the proposer in the next period selects the player with zero payoff as the coalition partner, and is thus able to take the whole dollar for herself. The same logic is also

\(^{36}\)Assume that there are \( M \) symmetrically dispersed industries such that \( \frac{n_1}{N} = \ldots = \frac{n_{M-1}}{N} \). To obtain majority, the support of the \( \frac{M-1}{2} \) industries are required besides the industry the proposer belongs. Then, the respective \( \text{ex post} \) tariffs turn out to be:

\[
\begin{align*}
\tau_i &= \frac{M+1}{2M} \theta - Q_i, \text{ for the proposer industry} \\
\tau_j &= \frac{1}{M} \theta - Q_j, \text{ for the } \frac{M-1}{2} \text{ partner industries} \\
\tau_k &= -Q_k, \text{ for the } \frac{M-1}{2} \text{ remaining industries.}
\end{align*}
\]

We can easily see that as \( M \) increases, the \( \text{ex post} \) tariffs obtained by the proposer and the coalition partners decrease. The \( \text{ex ante} \) tariffs decrease as well since they are equal to what coalition partners get adjusted for the total industry output (i.e., the second term in the coalition partner’s tariff).
at work in our model. In every period, one industry, say industry $i$, will be left out of the winning coalition, getting a tariff $\tau_i = -Q_i$, and having the lowest status quo payoff in the following period. Thus, if a legislator representing industry $j \neq i$ becomes the proposer in the following period, she chooses industry $i$ as her coalition partner, and is able to appropriate higher gains. After some time in the game, whoever is the proposer (say industry $i$) will propose $\tau^{U_i}$ (unconstrained maximization tariffs) and it will be accepted. This result is true irrespective of the discount factor and the status quo prices. However, although an industry is able to achieve its first-best when its representative becomes the proposer, it receives the worst possible payoff in the remaining scenarios. On average, it actually does worse relative to when bargaining ends once an agreement is reached.\(^{37}\) Hence, if we add an initial stage to our model where players can decide whether to play the game once or continuously, they will choose to play once. When $\delta > \frac{1}{2}$, on the other hand, legislators will agree on $\tau^{U_i}$ after a few periods. In this case, the expected payoffs remain the same as in our model.

The third point is about the discount factor. For analytical convenience, we have considered the limiting case in which $\delta$ approaches 1. In the context of a Baron-Ferejohn divide-the-dollar game with asymmetric recognition probabilities (as in our paper), Eraslan (2002) shows that an SSPE with fully mixed strategies does not exist when $\delta$ is below a certain threshold. This is also true in our game. When $\delta < 1$, depending on the values of $\left(\frac{n_i}{N}, \frac{n_j}{N}, \frac{n_k}{N}\right)$ and $(\tau^*_1 + Q_1, \tau^*_2 + Q_2, \tau^*_3 + Q_3)$, one or more industries may use pure strategies in choosing their coalition partners. For instance, when $\frac{n_i}{N} = \frac{n_j}{N} = \frac{n_k}{N} = \frac{1}{3}$, the industry with the highest $\tau^*_i + Q_i$ may never be chosen as a coalition partner if $\delta$ is sufficiently low.

\(^{37}\)Once the game converges to a stationary stage in which the proposer is able to achieve its first-best, industry $i$’s per-period continuation payoff becomes:

$$v_i = w_i(\tau^*) + \theta[(\frac{n_i}{N} \theta - Q_i) - \tau^*_i] - \frac{\theta^2}{2} - \sum_{l=i,j,k} \left[ (\tau^*_l + Q_l)^2 \right].$$

Since $\frac{n_i}{N} \leq \frac{1}{2}$, this is less than what industry $i$ expects in our game:

$$v_i = w_i(\tau^*) + \theta[(\frac{\theta}{3} - Q_i) - \tau^*_i] - \frac{5\theta^2}{9} - \sum_{l=i,j,k} \left[ (\tau^*_l + Q_l)^2 \right].$$
Similarly, when \( \tau_i^* + Q_1 = \tau_2^* + Q_2 = \tau_3^* + Q_3 = \tau^* + Q \), the industry with the highest \( n \) may never be chosen as a coalition partner if \( \delta \) lies below a threshold.\(^{38}\) However, in both cases our qualitative results would remain true. In particular, the ranking of welfare gains remains the same; i.e., the industry with the lowest \( \tau_i^* + Q_i \) does the best while the one with the highest \( \tau_i^* + Q_i \) does the worst. When \( \tau_1^* + Q_1 = \tau_2^* + Q_2 = \tau_3^* + Q_3 = \tau^* + Q \), all industries are equally well off unless one industry is sufficiently dispersed and \( \delta \) is sufficiently low so that it is never chosen as a coalition partner. In this case, that industry does better than others (namely, the benefit of being the proposer with high probability outweighs the loss from being excluded from the coalition). If the expected welfare gains are positive (negative), then they would decrease (increase) with a lower \( \delta \), eventually converging to zero as \( \delta \) goes to zero.

The final point is about the productivity parameter. Again, for analytical convenience, we have assumed that all industries have the same productivity. Once we assume different productivity for each industry, the model is not analytically tractable. Therefore, we solve the model numerically. In this case, there are too many moving parts in the model and it is hard to control all of them. To simplify matters, we assume that total industry output is the same for each sector. Let us define a vector \( \Theta = (\theta_1, \theta_2, \theta_3) \). We start from \( \Theta = (1, 1, 1) \) and then increase the productivity of industry 1 by increasing \( \theta_1 \) while keeping \( \theta_2, \theta_3 \) and total output (hence import penetration) constant. In this case, the \( \text{ex post} \) tariff levels depend on the identity of the proposer. However, we can say that among all the industries that are proposers, the higher productivity sector obtains the highest \( \text{ex post} \) tariff. The same result also holds among all the partner industries. On the other hand, the \( \text{ex ante} \) expected tariff level is smaller for the higher productivity industry, since as productivity of industry \( i \) increases, the probability that industry \( i \) is included in any coalition goes down in order to prevent industry \( i \)'s continuation payoff to increase further. In fact, it is easy to construct

\(^{38}\)It can be shown that when \( \tau^* + Q \) is in between \( \left[ \frac{\theta_i}{3}(1 - \frac{\sqrt{6}}{3}), \frac{\theta_i}{3}(1 + \frac{\sqrt{6}}{3}) \right] \), for any value of \( \delta \in (0, 1) \) there is always randomization as long as \( 0 \leq n_i \leq \frac{1}{2}, \forall i \). On the other hand, if \( \tau^* + Q \) is outside of the interval defined above, there will be randomization as long as each \( n_i \in [n, \bar{n}] \), where \( n > 0 \) and \( \bar{n} < \frac{1}{2} \) are endogenously determined as a function of \( (\tau^* + Q, \delta, \theta) \).
examples in which the \textit{ex ante} welfare change for the more productive industry is the worst. The only study we are aware of that focusses on the effect of productivity on protection is Karacaovali (2011), who shows that in Colombian data more productive industries \textit{ceteris paribus} receive more protection. It would be of interest to take our findings to that sort of data. One possibility would be to look at the interaction between industry productivity and membership in the legislative coalition (which could be proxied with legislative voting history). Membership in a legislative coalition has a positive effect on an industry’s tariff in our model, but the effect is larger if its productivity is higher.

5 Conclusion

We have developed a model of legislative trade policy-making in a setting of distributive politics. A small open economy has many districts, each one of which is associated with a particular industry. Thus, there is a conflict among districts hinged on industry attachment. Trade policy is determined collectively in the legislature as a result of bargaining among legislators, each of whom seeks to serve the interests of the district she represents. The legislative process is modeled as a multilateral sequential bargaining game \textit{à la} Baron and Ferejohn (1989).

Our analysis has three characteristics that are distinct from existing studies; (1) In addition to the usual factors accounted for in empirical work, the resulting trade policy depends on the identity of the agenda setter; (2) The congressional bargaining generally has an equilibrium in mixed strategies due to its dynamic nature; and (3) Because of the uncertainty about the future, strategic voting can lead a legislator to vote for a proposal that will make her district worse off compared to the \textit{status quo}.

In short, our model is dynamic and considers a parliamentary setting that stresses the importance of institutional structure on trade-policy formation. Furthermore, it is rich enough to encompass the findings of the existing literature as well as to incorporate new elements to them by analyzing the effects of dynamic, non-cooperative congressional bargaining.

In our model, we have considered the legislature as an ultimate decision-making body
without any outside interference. One might alternatively consider an executive with veto power, such as a President. In such a situation, even though the President does not have a decision-making authority, she can veto some proposals that are not in agreement with her own agenda. For example, in the United States, legislators come from plurality elections in small districts whereas the President is elected in national elections. The difference in constituents of legislatures and the Presidency could plausibly affect the preferences and goals that each brings to congressional bargaining. This extension is outside the scope of this paper, however.\textsuperscript{39}

\section*{Appendix}

\textbf{Proof of Proposition 2.} When a legislator representing industry \(i\) is selected as the proposer and chooses industry \(j \neq i\) as the coalition partner, we denote the chosen tariffs as \(\tau = (\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k)\) where, \(\tau_{ij}^i\) is the tariff industry \(i\) gets, \(\tau_{ij}^j\) is the tariff industry \(j\) gets and \(\tau_{ij}^k\) is the tariff industry \(k \neq i, j\) gets.

Now, suppose a legislator representing industry \(i\) is selected as the proposer and she chooses industry \(j \neq i\) as the coalition partner. Her maximization problem is

\[
\max_{\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k} w_i(\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k) \quad \text{s.t.} \quad w_j(\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k) \geq (1 - \delta)w_j(\tau^s) + \delta v_j.
\]

Maximizing per capita welfare \(w_i(\tau)\) is equivalent to maximizing \(w_i(\tau) - w_i(\tau^s)\). Furthermore, in the limit as \(\delta \to 1\), the constraint can be rewritten as \(w_j(\tau) \geq v_j\). Hence, the maximization problem becomes

\[
\max_{\tau} w_i(\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k) - w_i(\tau^s) \quad \text{s.t.} \quad w_j(\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k) \geq v_j,
\]

where

\[
\begin{align*}
w_i(\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k) &= w_i(\tau^s) + \frac{\theta(\tau_i - \tau_i^s) - \frac{1}{2} \sum_{l=i,j,k} [(\tau_l + Q_l)^2 - (\tau_l^s + Q_l)^2]}{2} , \\
w_j(\tau_{ij}^i, \tau_{ij}^j, \tau_{ij}^k) &= w_j(\tau^s) + \frac{\theta(\tau_j - \tau_j^s) - \frac{1}{2} \sum_{l=i,j,k} [(\tau_l + Q_l)^2 - (\tau_l^s + Q_l)^2]}{2}
\end{align*}
\]

\textsuperscript{39}We explore this possibility in a companion paper, Celik, Karabay and McLaren (2010).
The Lagrangian can be expressed as
\[
L(\tau_{ij}, \tau_{ij}, \tau_{ij}) = w_i(\tau_{ij}, \tau_{ij}, \tau_{ij}) + \lambda^{ij}(w_j(\tau_{ij}, \tau_{ij}, \tau_{ij}) - v_j),
\]
where \(\lambda^{ij}\) is the Lagrange multiplier when a legislator representing industry \(i\) is selected as the proposer and she chooses industry \(j \neq i\) as the coalition partner. It represents the cost to the proposing legislator of obtaining the additional votes needed to pass the proposal.

The first-order conditions, after simplification, are
\[
\tau_{ij} = \frac{\theta}{1 + \lambda^{ij}} - Q_i,
\]
\[
\tau_{ij} = \frac{\lambda^{ij} \theta}{1 + \lambda^{ij}} - Q_j,
\]
\[
\tau_{ij} = -Q_k.
\]

We first show that, in an SSPE in which all proposers employ mixed strategies in choosing their coalition partners, the value of \(\lambda^{ij}\) is independent of the identity of the proposer and of the coalition partner, i.e., \(\lambda^{ij} = \lambda\) for all \(i \neq j, i, j = 1, 2, 3\). This follows from the following two observations. First, a legislator would employ a mixed strategy in choosing a coalition partner only when the \textit{ex post} payoff her district enjoys is the same under each alternative. In other words, when a legislator representing industry \(i\) is selected as the proposer, she randomly picks an industry as a coalition partner if, for all \(i \neq j \neq k\),
\[
w_i(\tau_{ij}, \tau_{ij}, \tau_{ij}) = w_i(\tau_{ik}, \tau_{ik}, \tau_{ik})
\]
\[
\Leftrightarrow \theta \tau_{ij} - \frac{1}{2} \sum_{l=i,j,k} \left[ (\tau_{il} + Q_l)^2 - (\tau_{il} + Q_l)^2 \right] = \theta \tau_{ik} - \frac{1}{2} \sum_{l=i,j,k} \left[ (\tau_{il} + Q_l)^2 - (\tau_{il} + Q_l)^2 \right].
\]

Using the equilibrium values of \((\tau_{ij}, \tau_{ij}, \tau_{ij})\) and \((\tau_{ik}, \tau_{ik}, \tau_{ik})\), we have
\[
\frac{\theta^2}{1 + \lambda^{ij}} - \frac{1}{2} \left[ \frac{(1 + \lambda^{ij})^2}{(1 + \lambda^{ij})^2} \right] = \frac{\theta^2}{1 + \lambda^{ik}} - \frac{1}{2} \left[ \frac{(1 + \lambda^{ik})^2}{(1 + \lambda^{ik})^2} \right].
\]

It is easy to see that this is possible only if \(\lambda^{ij} = \lambda^{ik}\). Second, when industry \(j\) is chosen as a coalition partner, the \textit{ex post} welfare it is offered would be independent of the identity
of the proposer, because whoever is the proposer always offers an \textit{ex post} welfare of $v_j$ to this industry, otherwise the proposal is rejected. Thus, for any $i \neq j \neq k$,

$$w_j(\tau_i^{ij}, \tau_j^{ij}, \tau_k^{ij}) = w_j(\tau_i^{kj}, \tau_j^{kj}, \tau_k^{kj})$$

$$\Leftrightarrow \theta \tau_j^{ij} - \frac{1}{2} \sum_{l=i,j,k} \left[ (\tau_l^{ij} + Q_l)^2 - (\tau_l^s + Q_l)^2 \right] = \theta \tau_j^{kj} - \frac{1}{2} \sum_{l=i,j,k} \left[ (\tau_l^{kj} + Q_l)^2 - (\tau_l^s + Q_l)^2 \right].$$

Using the equilibrium values of $(\tau_i^{ij}, \tau_j^{ij}, \tau_k^{ij})$ and $(\tau_i^{kj}, \tau_j^{kj}, \tau_k^{kj})$, we have

$$\frac{\lambda^{ij} \theta^2}{1 + \lambda^{ij}} - \frac{1}{2} \left[ \frac{\left(1 + (\lambda^{ij})^2\right) \theta^2}{(1 + \lambda^{ij})^2} \right] = \frac{\lambda^{kj} \theta^2}{1 + \lambda^{kj}} - \frac{1}{2} \left[ \frac{\left(1 + (\lambda^{kj})^2\right) \theta^2}{(1 + \lambda^{kj})^2} \right].$$

Again, this is possible only if $\lambda^{ij} = \lambda^{kj}$. Together with the earlier observation, $\lambda^{ij} = \lambda^{kj} = \lambda^k$, which implies that $\lambda^{ij} = \lambda$ for all $i \neq j, i, j = 1, 2, 3$. Next, we find the equilibrium value of $\lambda$ in an SSPE in which all proposers employ mixed strategies in choosing their coalition partners. We first write down the equilibrium \textit{ex post} per capita welfare in three distinct cases.

(i) when the districts that produce good $j$ are selected as the proposer:

$$w_j^{\text{proposer}} = w_j(\tau^s) + \left[ \frac{\theta^2}{1 + \lambda} - \theta (\tau^s_j + Q_j) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (\tau_l^s + Q_l)^2 \right) \right].$$

(ii) when the districts that produce good $j$ are selected as a coalition partner:

$$w_j^{\text{partner}} = w_j(\tau^s) + \left[ \frac{\lambda \theta^2}{1 + \lambda} - \theta (\tau^s_j + Q_j) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (\tau_l^s + Q_l)^2 \right) \right].$$

(iii) when the districts that produce good $j$ are left outside the coalition:

$$w_j^{\text{outside}} = w_j(\tau^s) + \left[ -\theta (\tau^s_j + Q_j) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (\tau_l^s + Q_l)^2 \right) \right].$$

We next express the equilibrium continuation welfare of a district on a per capita basis. To do so, we need to introduce randomization probabilities. Let $s_{ij}$ denote the probability
that a legislator representing a district that produces good $i$ chooses the districts producing
good $j$ as a coalition partner. Then, $v_j$ can be expressed as

$$
v_j = \frac{n_j}{N}[s_{ji}w_{ij}^{\text{proposer}} + (1 - s_{ji})w_{ij}^{\text{proposer}] + \frac{n_i}{N}[s_{ij}w_{ij}^{\text{partner}} + (1 - s_{ij})w_{ij}^{\text{outside}] + \frac{n_k}{N}[s_{kj}w_{kj}^{\text{partner}} + (1 - s_{kj})w_{kj}^{\text{outside}]}. \quad (A.1)
$$

After simplification, this becomes

$$
v_j = w_j(\tau^s) + \frac{\theta^2}{1 + \lambda} \left( \frac{n_j}{N} + \left( \frac{s_{ij}n_i}{N} + s_{kj}n_k/N \right) \lambda \right) - \theta(\tau^s + Q_j) - \frac{1}{2} \left( \frac{1 + \lambda^2}{(1 + \lambda)^2} \theta^2 - \sum_{l=i,j,k} \left( \tau^s_i + Q_l \right)^2 \right). \quad (A.2)
$$

Next, observe that the maximization problem implies $w_j^{\text{partner}} = v_j$ (since the constraint
is binding in equilibrium). Hence, it must be true that

$$
\sum_{j=1}^{3} w_j^{\text{partner}} = \sum_{j=1}^{3} v_j.
$$

Also note that

$$
\sum_{j=1}^{3} \left( s_{ij} \frac{n_i}{N} + s_{kj} \frac{n_k}{N} \right) = \left( s_{12} \frac{n_1}{N} + s_{32} \frac{n_3}{N} \right) + \left( s_{13} \frac{n_1}{N} + s_{23} \frac{n_2}{N} \right) + \left( s_{21} \frac{n_2}{N} + s_{31} \frac{n_3}{N} \right)
$$

$$
= \left( s_{12} + s_{13} \right) \frac{n_1}{N} + \left( s_{21} + s_{23} \right) \frac{n_2}{N} + \left( s_{31} + s_{32} \right) \frac{n_3}{N}
$$

$$
= \frac{n_1 + n_2 + n_3}{N} = 1.
$$

The condition $\sum_{j=1}^{3} w_j^{\text{partner}} = \sum_{j=1}^{3} v_j$ can now be expressed as

$$
\frac{3\lambda\theta^2}{1 + \lambda} - \theta \sum_{j=1}^{3} \left( \tau^s_j + Q_j \right) - \frac{3}{2} \left( \frac{1 + \lambda^2}{(1 + \lambda)^2} \theta^2 - \sum_{l=i,j,k} \left( \tau^s_i + Q_l \right)^2 \right)
$$

$$
= \frac{\theta^2}{1 + \lambda} (1 + \lambda) - \theta \sum_{j=1}^{3} \left( \tau^s_j + Q_j \right) - \frac{3}{2} \left( \frac{1 + \lambda^2}{(1 + \lambda)^2} \theta^2 - \sum_{l=i,j,k} \left( \tau^s_i + Q_l \right)^2 \right)
$$

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\[ \Leftrightarrow \lambda = \frac{1}{2}. \]

So, the value of \( \lambda \) can be determined without the knowledge of the randomization probabilities. Plugging the equilibrium value of \( \lambda \) into the tariffs we found earlier gives

\[
\begin{align*}
\tau_i &= 2\theta - Q_i, \\
\tau_j &= \theta - Q_j, \\
\tau_k &= -Q_k.
\end{align*}
\]

The final step of the proof is to show that there is an interior solution to all of the randomization probabilities (this is what we assumed at the beginning of the proof). Since the continuation per-period, per-capita welfare is equal to \textit{ex post} welfare when chosen as a coalition partner (by the maximization problem), i.e., \( v_j = w_j^{\text{partner}} \), we have

\[
\frac{\theta^2}{1 + \lambda \left( \frac{n_j}{N} + \left( \frac{s_{ij} n_i}{N} + \frac{s_{kj} n_k}{N} \right) \lambda \right)} = \frac{\lambda \theta^2}{1 + \lambda}
\]

\[ \Leftrightarrow \frac{s_{ij} n_i}{N} + \frac{s_{kj} n_k}{N} = 1 - 2\frac{n_j}{N}. \]

For simplicity, let \( s_{12} = s_1, s_{23} = s_2 \) and \( s_{31} = s_3 \). Then,

\[
\begin{align*}
\frac{s_1 n_1}{N} + (1 - s_3) \frac{n_3}{N} &= 1 - 2\frac{n_2}{N}, \\
\frac{s_2 n_2}{N} + (1 - s_1) \frac{n_1}{N} &= 1 - 2\frac{n_3}{N}, \\
\frac{s_3 n_3}{N} + (1 - s_2) \frac{n_2}{N} &= 1 - 2\frac{n_1}{N}.
\end{align*}
\]

Note that these equations are linearly dependent (two of them imply the third), so we lose one degree of freedom. It is easy to check that, when \( \frac{n_3}{N} \leq \frac{n_2}{N} \leq \frac{n_1}{N} \leq \frac{1}{2} \), there is an interior solution in which \( s_i \in [0,1] \) for all \( i \). To see this, fix \( s_3 \) and express \( s_1 \) and \( s_2 \) in terms of \( s_3 \):

\[
s_1 = \frac{1 - 2\frac{n_2}{N} - (1 - s_3) \frac{n_3}{N}}{\frac{n_3}{N}},
\]

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\[ s_2 = 1 - \frac{1 - 2 \frac{n_1}{N} - s_3 \frac{n_3}{N}}{\frac{n_2}{N}}. \]

Any value of \( s_3 \in \left[ 0, \frac{1 - 2 \frac{n_1}{N}}{\frac{n_2}{N}} \right] \) yields \( s_1, s_2 \in [0, 1] \).

It is important to note that an industry may select its coalition partner with pure strategy. However, there are limitations. Feasible solutions (i.e., the solutions that satisfy \( s_i \in [0, 1] \) for all \( i \)) are

\[
(s_1, s_2, s_3) = \left( \frac{1 - 2 \frac{n_2}{N} - s_3 \frac{n_3}{N}}{\frac{n_1}{N}}, 1 - \frac{1 - 2 \frac{n_3}{N}}{\frac{n_2}{N}}, 0 \right),
\]

\[
(s_1, s_2, s_3) = \left( \frac{1 - s_1 \frac{n_1}{N} - \frac{n_2}{N}}{\frac{n_3}{N}}, 1, 1 - 2 \frac{n_3}{N} \frac{n_1}{N} \right).
\]

All three industries may use pure strategies only when \( \frac{n_1}{N} = \frac{n_2}{N} = \frac{n_3}{N} = \frac{1}{3} \). In this case, \( s_1 = s_2 = s_3 \) in all SSPE, so \( (s_1, s_2, s_3) = (1, 1, 1) \) and \( (s_1, s_2, s_3) = (0, 0, 0) \) are both possible. Similarly, when \( \frac{n_1}{N} = \frac{1}{2} \), industries 2 and 3 may use pure strategies. In fact, \( (s_1, s_2, s_3) = (1 - 2 \frac{n_2}{N}, 1, 0) \) is the unique SSPE in this case. Other than these two special cases, only industry 2 or industry 3 may select its coalition partner with pure strategy.

Finally, the continuation per-period welfare of a district on a per capita basis can be expressed, for all \( j = 1, 2, 3 \), as

\[
v_j = w_j(\tau^*) + \left[ \frac{\lambda \theta^2}{1 + \lambda} - \theta \left( \tau_j^s + Q_j \right) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (\tau_l^s + Q_l)^2 \right) \right].
\]

\[
= w_j(\tau^*) + \left[ \theta \left( \frac{\theta}{3} - (\tau_j^s + Q_j) \right) - \frac{\theta^2}{\theta^2} \sum_{l=i,j,k} (\tau_l^s + Q_l)^2 \right].
\]

**Proof of Proposition 3.** We will prove payoff uniqueness by showing that the randomization strategies that are not already accounted for in the proof of Proposition 2 cannot arise in an SSPE. Possible scenarios include all three industries or only two industries using pure strategies. We will eliminate all possibilities step by step. When only one industry uses pure strategy, the proof of Proposition 2 is perfectly applicable, so possible SSPE involve industry
2 choosing industry 3 with pure strategy, or vice versa. All other possibilities can be ruled out because they lead to non-interior solutions (i.e., $s_i \not\in [0,1]$ for some $i$).

Before proceeding with the proof, we would like to make an important observation that will prove very helpful. The equilibrium per capita welfare function of the proposer,

$$w^\text{proposer}_j = w^*_j(\tau^*) + \left[ \frac{\theta^2}{1 + \lambda} - \theta \left( \tau^*_j + Q_j \right) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (\tau^*_l + Q_l)^2 \right) \right],$$

is strictly decreasing in $\lambda$ for all $\lambda > 0$. This means that if $\lambda^{ik} > \lambda^{kj}$, then we must have $s_{ik} = 0$ because, otherwise, industry $i$ can profitably deviate by mimicking industry $k$ and selecting industry $j$ as a partner rather than industry $k$.

**Observation 1.** The following configuration cannot arise in an SSPE: $s_{ik} = s_{jk} = 1$ for $i \neq j \neq k$.

This is the case when one industry is selected as a coalition member with pure strategy by each one of the other two industries. Intuitively, this puts industry $k$ in a veto-player position which enables it to achieve its first-best. This, in turn, leads to a profitable deviation by each one of the other two industries. To see this, we follow the same steps as in the proof of Proposition 2. Under the strategies $s_{ik} = s_{jk} = 1$, both industry $i$ and $j$ give industry $k$ its continuation payoff, so,

$$\lambda^{ik} = \lambda^{jk}.$$

For industry $k$ to randomize between the other two industries according to $s_{ki} \in [0,1]$, it must also be true that

$$\lambda^{ki} = \lambda^{kj}.$$

Let $\lambda^{ik} = \lambda^{jk} = \mu$ and $\lambda^{ki} = \lambda^{kj} = \lambda$. Then, for industries $i$ and $j$ not to deviate from $s_{ik} = s_{jk} = 1$, we must have $\mu \leq \lambda$. The equilibrium *ex post* welfare of industry $k$ as the proposer and as a partner can be expressed as

$$w^\text{proposer}_k = w_k(\tau^*) + \left[ \frac{\theta^2}{1 + \lambda} - \theta \left( \tau^*_k + Q_k \right) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (\tau^*_l + Q_l)^2 \right) \right],$$
\[ w_k^{\text{partner}} = w_k(\tau^*) + \left[ \frac{\mu \theta^2}{1 + \mu} - \theta (\tau_{k}^* + Q_k) - \frac{1}{2} \left( \frac{(1 + \mu^2) \theta^2}{(1 + \mu)^2} - \sum_{l=i,j,k} (\tau_l^* + Q_l)^2 \right) \right]. \]

By (A.1), then,

\[ v_k = w_k(\tau^*) + \frac{n_k \theta^2}{1 + \lambda} + \frac{(n_i + n_j) \mu \theta^2}{1 + \mu} - \theta (\tau_{k}^* + Q_k) \]

\[ - \frac{1}{2} \left( \frac{n_k (1 + \lambda^2) \theta^2}{N (1 + \lambda)^2} + \frac{(n_i + n_j) (1 + \mu^2) \theta^2}{N (1 + \mu)^2} - \sum_{l=i,j,k} (\tau_l^* + Q_l)^2 \right). \]

This leads, by the constraint \( w_k^{\text{partner}} = v_k \), to

\[ \frac{\mu \theta^2}{1 + \mu} - \frac{1}{2} \left( \frac{(1 + \mu^2) \theta^2}{(1 + \mu)^2} \right) = \frac{n_k \theta^2}{1 + \lambda} + \frac{(n_i + n_j) \mu \theta^2}{1 + \mu} - \frac{1}{2} \left( \frac{n_k (1 + \lambda^2) \theta^2}{N (1 + \lambda)^2} + \frac{(n_i + n_j) (1 + \mu^2) \theta^2}{N (1 + \mu)^2} \right) \]

\[ \Leftrightarrow \frac{\mu \theta^2}{1 + \mu} - \frac{1}{2} \left( \frac{(1 + \mu^2) \theta^2}{(1 + \mu)^2} \right) = \frac{\theta^2}{1 + \lambda} - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} \right). \]

This is possible only when \( \mu = 1/\lambda \). Together with the earlier condition that \( \mu \leq \lambda \), this requires \( \lambda \geq 1 \). Repeating the same steps for industry \( i \), we have

\[ w_i^{\text{partner}} = w_i(p^*) + \left[ \frac{\lambda \theta^2}{1 + \lambda} - \theta (\tau_{i}^* + Q_i) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau_l^*)^2 \right) \right], \]

\[ v_i = w_i(\tau^*) + \frac{n_i \theta^2}{1 + \mu} + \frac{n_k s_{ki} \lambda \theta^2}{1 + \lambda} - \theta (\tau_{i}^* + Q_i) \]

\[ - \frac{1}{2} \left( \frac{(n_i + n_j) (1 + \lambda^2) \theta^2}{N (1 + \lambda)^2} + \frac{n_k (1 + \lambda^2) \theta^2}{N (1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau_l^*)^2 \right) \]

\[ = w_i(\tau^*) + \frac{(n_i + n_k s_{ki}) \lambda \theta^2}{1 + \lambda} - \theta (\tau_{i}^* + Q_i) \]

\[ - \frac{1}{2} \left( \frac{(n_i + n_j + n_k) (1 + \lambda^2) \theta^2}{N (1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau_l^*)^2 \right), \]

where the last line uses the fact that \( \mu = 1/\lambda \). By the constraint \( w_i^{\text{partner}} = v_i \), and \( n_i + n_j + n_k = 1 \), then,

\[ \frac{\lambda \theta^2}{1 + \lambda} - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} \right) = \frac{(n_i + n_k s_{ki}) \lambda \theta^2}{1 + \lambda} - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} \right), \]

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which can be satisfied only when \( \lambda = 0 \). Thus, we reach a contradiction.

**Observation 2.** Unless \( \frac{n_i}{N} = \frac{n_j}{N} = \frac{n_k}{N} \), the following configuration cannot arise in an SSPE: \( s_{ij} = s_{jk} = s_{ki} = 1 \) for \( i \neq j \neq k \).

Under this configuration, each industry is selected as a coalition member by one of the other two industries. As in the proof of Observation 1, for each industry to play according to \( s_{ij} = s_{jk} = s_{ki} = 1 \) and not to deviate to choosing another partner, we must have

\[
\lambda^{ij} \leq \lambda^{jk}, \\
\lambda^{jk} \leq \lambda^{ki}, \\
\lambda^{ki} \leq \lambda^{ij}.
\]

These three inequalities can be satisfied only when \( \lambda^{ij} = \lambda^{jk} = \lambda^{ki} = \lambda \).

Next, using (A.1), we express the continuation payoff of each industry as follows.

\[
v_i = w_i(\sigma^s) + \left[ \frac{(n_i + n_k \lambda)}{1 + \lambda} - \theta (\tau^s_i + Q_i) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau^s_l)^2 \right) \right],
\]

\[
v_j = w_j(\sigma^s) + \left[ \frac{(n_j + n_k \lambda)}{1 + \lambda} - \theta (\tau^s_j + Q_j) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau^s_l)^2 \right) \right],
\]

\[
v_k = w_k(\sigma^s) + \left[ \frac{(n_k + n_i \lambda)}{1 + \lambda} - \theta (\tau^s_k + Q_k) - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau^s_l)^2 \right) \right].
\]

By the constraint \( w^\text{partner}_l = v_l \) for each \( l = i, j, k \), then,

\[
\lambda = \frac{n_i}{N} + \frac{n_k}{N} \lambda, \\
\lambda = \frac{n_j}{N} + \frac{n_i}{N} \lambda, \\
\lambda = \frac{n_k}{N} + \frac{n_j}{N} \lambda.
\]

Summing each side and using \( \frac{n_i + n_j + n_k}{N} = 1 \) gives

\[\lambda = \frac{1}{2}.\]
Plugging the value of $\lambda$ into the above equations leads to

$$\frac{n_i}{N} + \frac{n_k}{2} = \frac{n_j}{N} + \frac{n_j}{2} = \frac{n_k}{N} + \frac{n_j}{2} = \frac{1}{2}. $$

But this is possible only when $n_i = n_j = n_k = \frac{1}{3}$.

**Observation 3.** The following configuration cannot arise in an SSPE: $s_{ij} = s_{jk} = 1$ and $s_{ki} \in (0,1)$ for $i \neq j \neq k$.

Under this configuration, industry $i$ is selected as a coalition member less often than the others, which lowers its equilibrium continuation payoff. This, in turn, induces industry $k$ to choose industry $i$ with pure strategy. Note that we have assumed $s_{ki} \in (0,1)$ since $s_{ki} = 0$ can be ruled out by Observation 1 while $s_{ki} = 1$ can be ruled out by Observation 2.

For each industry to play according to $s_{ij} = s_{jk} = 1$ and $s_{ki} \in (0,1)$, we must have

$$\chi^{ij} \leq \chi^{jk},$$

$$\chi^{jk} \leq \chi^{ki},$$

$$\chi^{ki} = \chi^{kj},$$

$$\chi^{kj} = \chi^{ij}.$$  

These four expressions can again be satisfied only when $\chi^{ij} = \chi^{jk} = \chi^{ki} = \chi^{kj} = \lambda$.

Next, using (A.1), we express the continuation payoff of each industry:

$$v_i = w_i(\tau^*) + \left[ \frac{\alpha_i + \alpha_k s_{ki} \lambda}{1 + \lambda} \theta^2 + \theta (\tau_i^* + Q_i) - \frac{1}{2} \left( \frac{(1 + \lambda \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau_l^*)^2 \right) \right],$$

$$v_j = w_j(\tau^*) + \left[ \frac{\alpha_j + \alpha_k s_{kj} \lambda}{1 + \lambda} \theta^2 + \theta (\tau_j^* + Q_j) - \frac{1}{2} \left( \frac{(1 + \lambda \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau_l^*)^2 \right) \right],$$

$$v_k = w_k(\tau^*) + \left[ \frac{\alpha_k + \alpha_j s_{ij} \lambda}{1 + \lambda} \theta^2 + \theta (\tau_k^* + Q_k) - \frac{1}{2} \left( \frac{(1 + \lambda \theta^2}{(1 + \lambda)^2} - \sum_{l=i,j,k} (Q_l + \tau_l^*)^2 \right) \right].$$

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By the constraint $u_l^{\text{partner}} = v_l$ for each $l = i, j, k$, then,

$$
\lambda = \frac{n_i}{N} + \frac{n_k}{N}s_{ki}\lambda,
$$

$$
\lambda = \frac{n_j}{N} + \left(\frac{n_i}{N} + \frac{n_k}{N}s_{kj}\right)\lambda,
$$

$$
\lambda = \frac{n_k}{N} + \frac{n_j}{N}\lambda.
$$

Summing each side and using $\frac{n_i+n_j+n_k}{N} = 1$ gives

$$
\lambda = \frac{1}{2}.
$$

Using $\lambda = \frac{1}{2}$, $s_{ki} = \frac{1-2n_i}{N}$ from the first of the above equations. Plugging this expression into the second equation and using $\frac{n_i+n_j+n_k}{N} = 1$ gives

$$
\frac{n_j}{N} + 2\frac{n_i}{N} = 1.
$$

However, the third equation above implies

$$
2\frac{n_k}{N} + \frac{n_j}{N} = 1,
$$

so it must be that $\frac{n_i}{N} = \frac{n_j}{N} = \frac{n_k}{N} = \frac{1}{3}$. But then $s_{ki} = \frac{1-2n_i}{N} = 1$, which is a contradiction.

**Observation 4.** Unless $\frac{n_k}{N} = \frac{1}{2}$, the following configuration cannot arise in an SSPE: $s_{ij} = s_{ji} = 1$ and $s_{ki} \in (0, 1)$ for $i \neq j \neq k$.

Under this configuration, industry $k$ is never selected as a coalition member, which lowers its equilibrium continuation payoff. This, in turn, leads to a deviation by the other two industries. Note that we have assumed $s_{ki} \in (0, 1)$ since both $s_{ki} = 0$ and $s_{ki} = 1$ can be ruled out by Observation 1.

For each industry to play according to $s_{ij} = s_{ji} = 1$ and $s_{ki} \in (0, 1)$, we must have

$$
\lambda^{ki} = \lambda^{kj},
$$

$$
\lambda^{ki} = \lambda^{ji},
$$

$$
\lambda^{kj} = \lambda^{ij}.
$$
Thus, \( \lambda^{ij} = \lambda^{ji} = \lambda^{ki} = \lambda^{kj} = \lambda \).

Next, using (A.1), we express the continuation payoff of industries \( i \) and \( j \):

\[
v_i = w_i(\tau^s) + \left[ \frac{n_i}{N} \left( \frac{n_j}{N} + \frac{n_k}{N} s_{ki} \right) \lambda \right] \theta^2 - \theta(\tau_i^s + Q_i) - \frac{1}{2} \left( \frac{(1 + \lambda^2)}{(1 + \lambda)^2} \theta^2 - \sum_{l=i,j,k} (Q_l + \tau_l^s)^2 \right),
\]

\[
v_j = w_j(\tau^s) + \left[ \frac{n_j}{N} \left( \frac{n_i}{N} + \frac{n_k}{N} s_{kj} \right) \lambda \right] \theta^2 - \theta(\tau_j^s + Q_j) - \frac{1}{2} \left( \frac{(1 + \lambda^2)}{(1 + \lambda)^2} \theta^2 - \sum_{l=i,j,k} (Q_l + \tau_l^s)^2 \right).
\]

By the constraints \( w_l^{\text{partner}} = v_l \) for \( l = i, j \) (note that \( w_k^{\text{partner}} = v_k \) cannot be used here since industry \( k \) is never selected as a coalition member), then,

\[
\lambda = \frac{n_i}{N} + \left( \frac{n_j}{N} + \frac{n_k}{N} s_{ki} \right) \lambda,
\]

\[
\lambda = \frac{n_j}{N} + \left( \frac{n_i}{N} + \frac{n_k}{N} s_{kj} \right) \lambda.
\]

Summing each side and using \( \frac{n_i + n_j + n_k}{N} = 1 \) gives

\[
\lambda = \frac{n_i + n_j}{N}.
\]

Now, let us write down industry \( k \)'s continuation payoff:

\[
v_k = w_k(\tau^s) + \left[ \frac{n_k}{N} \theta^2 \right] \frac{1}{1 + \lambda} - \theta(\tau_k^s + Q_k) - \frac{1}{2} \left( \frac{(1 + \lambda^2)}{(1 + \lambda)^2} \theta^2 - \sum_{l=i,j,k} (Q_l + \tau_l^s)^2 \right).
\]

But then, industry \( i \) (same applies for industry \( j \), too) can deviate by selecting industry \( k \) as a coalition partner and offering a tariff vector:

\[
(\tau_i^{ik}, \tau_j^{ik}, \tau_k^{ik}) = \left( \frac{\theta}{1 + \frac{n_k}{N}} - Q_i, -Q_j, \frac{\frac{n_k}{N} \theta}{1 + \frac{n_k}{N}} - Q_k \right),
\]

in which case,

\[
w_i^{\text{proposer}} = w_i(\tau^s) + \left[ \frac{\theta^2}{1 + \frac{n_k}{N}} - \theta(\tau_i^s + Q_i) - \frac{1}{2} \left( \frac{(1 + \left( \frac{n_k}{N} \right)^2)}{(1 + \frac{n_k}{N})^2} \theta^2 - \sum_{l=i,j,k} (Q_l + \tau_l^s)^2 \right) \right]
\]

\[
w_k^{\text{partner}} = w_k(\tau^s) + \left[ \frac{n_k \theta^2}{1 + \frac{n_k}{N}} - \theta(\tau_k^s + Q_k) - \frac{1}{2} \left( \frac{(1 + \left( \frac{n_k}{N} \right)^2)}{(1 + \frac{n_k}{N})^2} \theta^2 - \sum_{l=i,j,k} (Q_l + \tau_l^s)^2 \right) \right].
\]

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Note that, unless \( \frac{n_k}{N} = \frac{1}{2} \), we have \( \frac{n_k}{N} < \lambda = \frac{n_i + n_j}{N} \). Also note that \( \frac{(1+\lambda^2)\theta^2}{(1+\lambda)^2} \) is increasing in \( \lambda \) for \( \lambda < 1 \). Hence, both industry \( i \) and \( k \) benefit. This is a contradiction to the initial configuration. ■

References


